Comparison of Results on the 2006 CEC Benchmark Function Set

J. J. Liang and P. N. Suganthan

School of Electrical and Electronic Engineering,
Nanyang Technological University
epnsugan@ntu.edu.sg

July, 2006

Evaluation Criteria

- **Problems:** 24 minimization problems with constraints
- Minimize $f(\vec{x}), \vec{x} = (x_1, x_2, ..., x_n)$
- Subject to
$$g_j(\vec{x}) \le 0, \quad for \quad j = 1, ..., q$$
$$h_j(\vec{x}) = 0, \quad for \quad j = q+1, ..., m$$
- Equality constraints are transformed into inequalities of the form
$$\left|h_j(\vec{x})\right| - \varepsilon \le 0, \quad for \quad j = q+1, ..., m$$
- In this special session ε is set to 0.0001.

- Reference:

J. J. Liang, T. P. Runarsson, E. Mezura-Montes, M. Clerc, P. N. Suganthan, C. A. Coello Coello & K. Deb,"Problem Definitions and Evaluation Criteria for the CEC 2006 Special Session on Constrained Real-Parameter Optimization", *Technical Report*, Nanyang Technological University, Singapore, Dec 2005. http://www.ntu.edu.sg/home/EPNSugan/

Evaluation Criteria

Nanyang Technological University

Function	n	Type of f	ρ	LI	NI	LE	NE	a
G1	13	quadratic	0.0111%	9	0	0	0	6
G2	20	nonlinear	99.8474%	0	2	0	0	1
G3	10	polynomial	0.0000%	0	0	0	1	1
G4	5	quadratic	52.1230%	0	6	0	0	2
G5	4	cubic	0.0000%	2	0	0	3	3
G6	2	cubic	0.0066%	0	2	0	0	2
G7	10	quadratic	0.0003%	3	5	0	0	6
G8	2	nonlinear	0.8560%	0	2	0	0	0
G9	7	polynomial	0.5121%	0	4	0	0	2
G10	8	linear	0.0010%	3	3	0	0	6
G11	2	quadratic	0.0000%	0	0	0	1	1
G12	3	quadratic	4.7713%	0	1	0	0	0
G13	5	nonlinear	0.0000%	0	0	0	3	3
G14	10	nonlinear	0.0000%	0	0	3	0	3
G15	3	quadratic	0.0000%	0	0	1	1	2
G16	5	nonlinear	0.0204%	4	34	0	0	4
G17	6	nonlinear	0.0000%	0	0	0	4	4
G18	9	quadratic	0.0000%	0	12	0	0	4

Evaluation Criteria

G19	15	nonlinear	33.4761%	0	5	0	0	-
G20	24	linear	0.0000%	0	6	2	12	-
G21	7	linear	0.0000%	0	1	0	5	6
G22	22	linear	0.0000%	0	1	8	11	-
G23	9	linear	0.0000%	0	2	3	1	-
G24	2	linear	79.6556%	0	2	0	0	2

n: the number of decision variables.

ρ=|***F***|/|***S***| : the estimated ratio between the feasible region and the search space.

LI : the number of linear inequality constraints,

NI : the number of nonlinear inequality constraints,

LE : the number of linear equality constraints.

NE : the number of nonlinear equality constraints.

a: the number of active constraints at x*.

Evaluation Criteria

- **Runs / problem:** 25 (total runs)
- **Max_FES:** 500,000

- **Feasible Rate** = (# of feasible runs) / total runs
- **Success Rate** = (# of successful runs) / total runs
- **Success Performance** = mean (FEs for successful runs)*(# of total runs) / (# of successful runs)
  - The above three quantities are computed for each problem separately.
  - **Feasible Run:** A run during which at least one feasible solution is found in Max_FES.
  - **Successful Run:** A run during which the algorithm finds a feasible solution $\vec{x}$ satisfying $f(\vec{x}) - f(\vec{x}^*) \leq 0.0001$

Algorithms

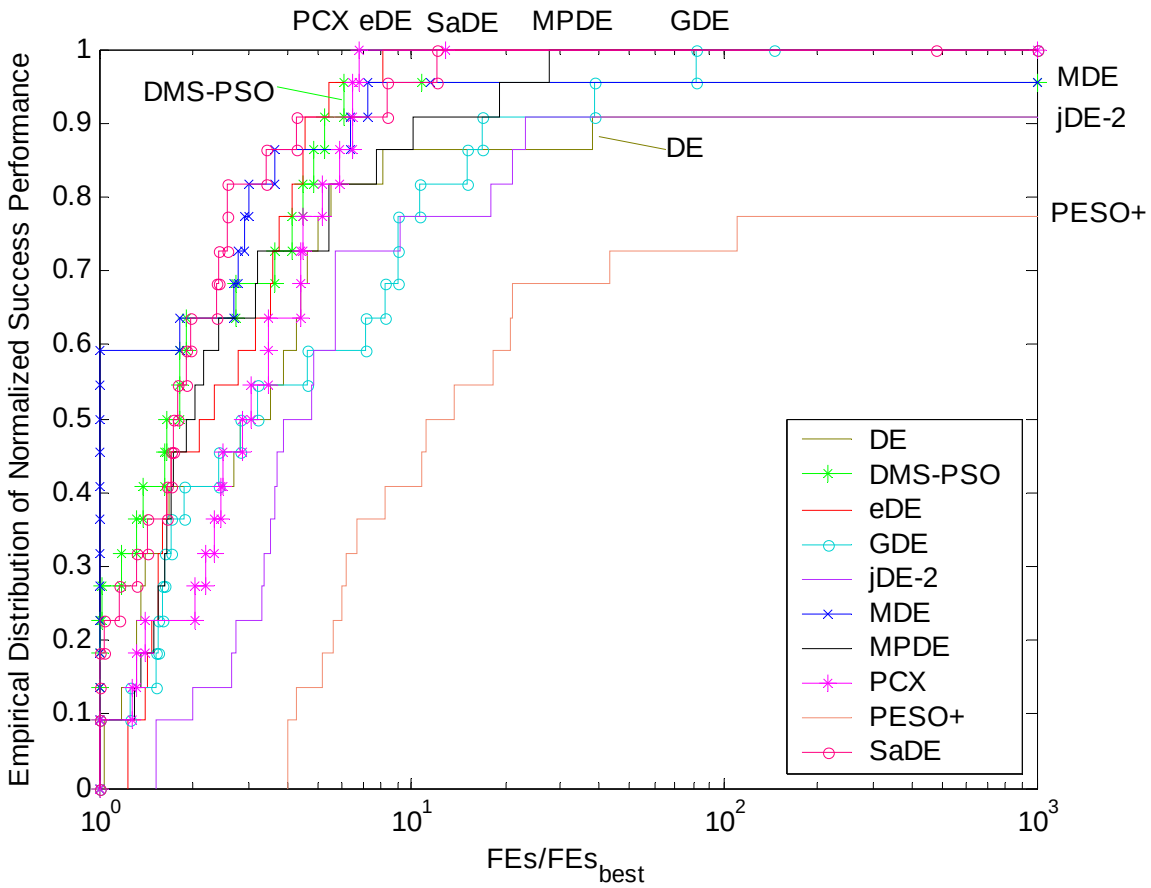
- DE Karin Zielinski & Rainer Laur (Constrained Single-…)
- DMS-PSO J. J. Liang & P. N. Suganthan (Dynamic Multi-Swarm...)
- ε _DE Tetsuyuki Takahama & Setsuko Sakai (Constrained ...)
- GDE Saku Kukkonen & Jouni Lampinen (Constrained …)
- jDE-2 Janez Brest & Viljem Zumer (Self-Adaptive ...)
- MDE Efrén Mezura-Montes, *et al* (Modified Differential….)
- MPDE M. Fatih Tasgetiren & P. N. Suganthan (A Multi-...)
- PCX Ankur Sinha, *et al* (A Population-Based, Parent ….)
- PESO+ Angel E. Munoz-Žavala *et al* (PESO+ for…)
- SaDE V. L. Huang *et al* (Selt-adaptive Differential…)

Algorithms' Parameters

Nanyang Technological University

DE	*NP*, *F*, *CR*
DMS-PSO	ω, c_1, c_2 , Vmax, *n*, *ns*, *R*, *L*, *L_FES*
ε_DE	*N*, *F*, *CR*, *Tc*, *Tmax*, *cp*, *Pg*, *Rg*, *Ne*
GDE	*NP*, *F*, *CR*
jDE-2	*NP*, *F*, *CR*, *k*, *l*
MDE	μ, *CR*, *Max_Gen*, λ, F_α, F_β,
MPDE	*F*, *CR*, *np1*, *np2*
PCX	*N* , λ, *r* (a different *N* is used for g02),
PESO+	ω , c_1, c_2 , *n*, *not sensitive to* ω , *c1*, *c2*
SaDE	*NP*, *LP*, *LS_gap*

Empirical Distribution of Normalized Success Performance



- Only the results of functions where at least one algorithm was successful at least once are used. (Thus g20 and g22 are not considered here)
- FEs here is Success Performance. $FEs_{\_best}$ is FEs of the best algorithm on the respective function.

Comparisons

Nanyang Technological University

Algorithms	Feasible Rate	Success Rate
DE	95.65%	78.09%
DMS-PSO	100%	90.61%
ε_DE	100%	95.65%
GDE	92.00%	77.39%
jDE-2	95.65%	80.00%
MDE	95.65%	87.65%
MPDE	94.96%	87.65%
PCX	95.65%	94.09%
PESO+	95.48%	67.83%
SaDE	100%	87.13%

* g20 is not considered here since it has no feasible solution

Comparisons

EAs \ FEs_{best}	g01	g02	g03	g04	g05	g06	g07	g08
	25115	96222	24861	15281	21306	5202	26578	918
DE	1.3304	1.4017	-	1.0461	5.0256	1.3731	3.5290	1.1830
DMS	1.3272	1.8201	1.0289	1.6625	1.3790	5.3126	1.0000	4.4928
ε_DE	2.3615	1.5571	3.5963	1.7156	4.5729	1.4189	2.7957	1.2407
GDE	1.6133	1.5543	143.8877	1.0000	9.0821	1.2501	4.6654	1.6002
jDE-2	2.0062	1.5163	-	2.6653	20.9724	5.6686	4.8064	3.5251
MDE	3.0011	1.0000	1.8096	2.7198	1.0000	1.0000	7.3069	1.0000
MPDE	1.7292	3.1694	1.0000	1.3666	10.1600	2.0327	2.1597	1.6498
PCX	2.1981	1.3292	1.4053	2.0279	4.4478	6.5015	4.4067	3.0784
PESO+	4.0427	4.2905	18.1268	5.2271	21.2267	10.8627	13.8191	6.6710
SaDE	1.0000	1.9107	12.0254	1.6430	3.4263	2.4118	1.0398	1.4412

*** Table entries: Success Performance FEs divided by FEs of the best algorithm**

Comparisons

Nanyang Technological University

EAs \ FEs_{best}	g09	g10	g11	g12	g13	g14	g15	g16
	16152	25520	3000	1308	21732	25220	10458	8730
DE	1.5976	4.6715	4.4600	3.9021	1.5976	2.7052	5.5429	1.3278
DMS	1.8237	1.0000	4.8750	4.1356	1.8237	1.0000	2.7634	6.1260
ε_DE	1.4315	4.1236	5.4733	3.1529	1.4315	4.4980	8.0528	1.4875
GDE	1.8716	3.2368	2.8200	2.4075	1.8716	9.1247	7.1605	1.5148
jDE-2	3.4001	5.7269	17.9760	4.8593	3.4001	3.8797	23.0812	3.6306
MDE	1.0000	6.4326	1.0000	1.0000	1.0000	11.5639	1.0000	1.0000
MPDE	1.3029	1.9055	7.7854	3.2401	1.3029	1.6937	19.1408	1.4963
PCX	2.8806	3.4886	12.8960	6.8502	2.8806	2.3488	4.4880	3.4817
PESO+	6.0391	110.8383	150.0333	6.1835	6.0391	-	43.0388	5.6174
SaDE	1.3278	1.7307	8.3703	1.9694	1.3278	1.7843	2.5818	1.7123

*** Table entries: Success Performance FEs divided by FEs of the best algorithm**

Comparisons

Nanyang Technological University

EAs \ FEs_{best}	g17	g18	g19	g21	g22	g23	g24
	26364	28261	21830	38217	-	129550	1794
DE	50.3891	2.8151	8.1186	4.2571	-	-	1.6856
DMS	-	1.1741	1.0000	3.6722	-	1.6251	10.8004
ε_DE	3.7498	2.0931	16.3239	3.5362	-	1.5497	1.6455
GDE	81.4890	16.9874	10.5489	15.1615	-	8.2081	1.7051
jDE-2	426.0602	3.6963	9.1548	3.3103	-	2.7592	5.6834
MDE	1.0000	3.6617	-	2.9455	-	2.7821	1.0000
MPDE	27.7422	1.5585	5.4180	5.4703	-	1.6261	2.4204
PCX	5.1627	2.4779	5.9403	1.0000	-	1.2900	6.4916
PESO+	-	8.2431	-	-	-	-	11.1371
SaDE	474.1314	1.0000	2.3896	4.2958	-	1.0000	2.5775

* Table entries: Success Performance FEs divided by FEs of the best algorithm

Rank

Algorithms	$\overline{f(x)}^{r}$	Feasible Rate	Success Rate	Success Performance	Final Rank
DE	7	4	8	6	7
DMS-PSO	3	1	3	3	2
ε_DE	1	1	1	4	1
GDE	10	10	9	8	9
jDE-2	8	4	7	9	8
MDE	5	4	4	1	3
MPDE	4	9	5	5	6
PCX	2	4	2	7	5
PESO+	9	8	10	10	9
SaDE	6	1	6	1	3

$$*Rank(f) = Rank(\sum_{i=1}^{n} Rank(f_i))$$

*$g20$ is not considered in the rank since it has no feaible solution

Final Rank

1st	**ε_DE**
2nd	**DMS-PSO**
3rd	**SaDE, MDE**
5th	**PCX**
6th	**MPDE**
7th	**DE**
8th	**jDE-2**
9th	**GDE, PESO+**