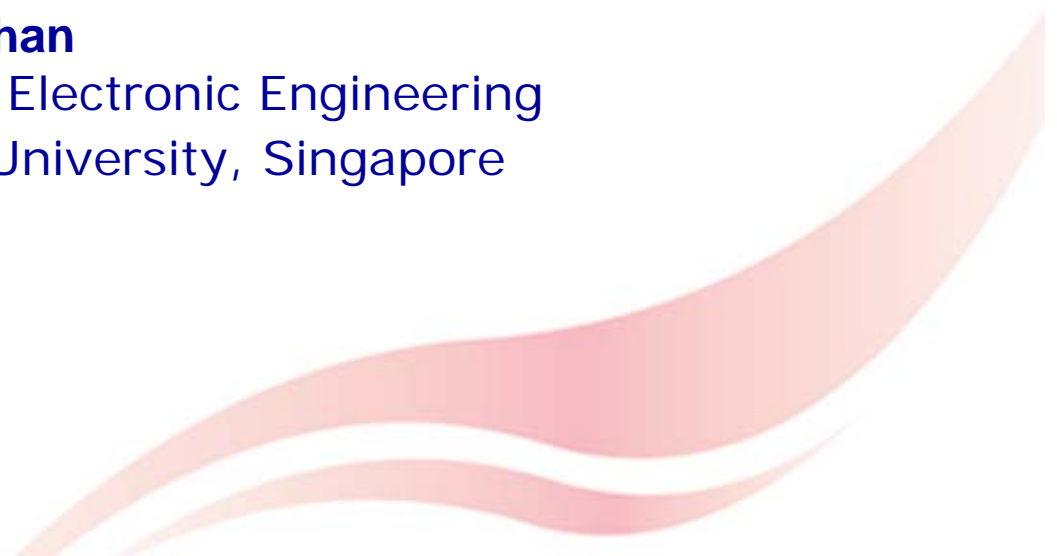# Performance Assessment on Multi-objective Optimization Algorithms

presented by

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Evaluation Criteria

- Quantitative performance measurements, **R indicator** and **Hypervolume difference to a reference set** are used as measures for the expected number of function evaluations to reach a target Pareto front.

- Invariance is the ability to generalize performance results. Possible invariances can include invariance against translation, scaling, rotation and so on.

- Each participant was asked to specify the followings on parameters settings
  – how many parameters of the algorithm need to be adjusted to the object function?
  – how many different settings were tested?
  – how many different settings were finally used?

References to Papers / Algorithms

- NSGAII_SBX: Sharma *et al.* "Hybridization of SBX Based NSGA-II and …"

- NSGAII_PCX: Kumar *et al.* "A Hybrid Multi-Objective Optimization …"

- GDE3: Kukkonen and Lampinen, "Performance Assessment of Generalized …"

- DEMOwSA: Zamuda *et al.* "Differential Evolution for Multiobjective …"

- MOSaDE: Huang *et al.,* "Multi-objective Optimization based on …"

- MO_DE: Zielinski and Laur, "Differential Evolution with Adaptive …"

- MO_PSO: Zielinski and Laur, "Adaptive Parameter Setting for …"

- MTS: Tseng and Chen, "Multiple Trajectory Search for …"

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Function Sets

- Three subsets
  - 2-objective functions (7)
  - 3-objective functions (6)
  - 5-objective functions (6)

- Comparison: Rank of the mean value of the metrics from 25 runs

M=2, Rank(R indicator)

FES=5000

	Total	1.OKA2	2.SYMPART	3.S_ZDT1	4.S_ZDT2	5.S_ZDT4	6.R_ZDT4	7.S_ZDT6
NSGA2_SBX	12	2	3	1	1	2	2	1
NSGA2_PCX	38	4	1	8	8	8	1	8
GDE3	29	1	7	4	4	5	4	4
DEMOwSA	45	3	8	6	7	7	7	7
MOSaDE	29	8	4	3	3	3	5	3
MO_DE	40	5	6	5	5	6	8	5
MO_PSO	35	7	2	7	6	4	3	6
MTS	24	6	5	2	2	1	6	2

FES=50000

	Total	1.OKA2	2.SYMPART	3.S_ZDT1	4.S_ZDT2	5.S_ZDT4	6.R_ZDT4	7.S_ZDT6
NSGA2_SBX	22	2	5	2	2	2	6	5
NSGA2_PCX	34	1	7	6	3	3	4	7
GDE3	17	3	2	4	4	4	3	2
DEMOwSA	33	4	4	3	6	7	2	3
MOSaDE	25	8	6	1	1	1	1	1
MO_DE	29	5	3	5	7	6	5	4
MO_PSO	51	7	1	8	8	8	7	8
MTS	41	6	8	7	5	5	8	6

FES=500000

	Total	1.OKA2	2.SYMPART	3.S_ZDT1	4.S_ZDT2	5.S_ZDT4	6.R_ZDT4	7.S_ZDT6
NSGA2_SBX	24	2	5	2	2	2	6	5
NSGA2_PCX	31	1	7	6	3	3	4	7
GDE3	22	3	2	4	4	4	3	2
DEMOwSA	29	4	4	3	6	7	2	3
MOSaDE	19	8	6	1	1	1	1	1
MO_DE	35	5	3	5	7	6	5	4
MO_PSO	47	7	1	8	8	8	7	8
MTS	45	6	8	7	5	5	8	6

M=2, Rank(Hypervolumn)

FES=5000

	Total	1.OKA2	2.SYMPART	3.S_ZDT1	4.S_ZDT2	5.S_ZDT4	6.R_ZDT4	7.S_ZDT6
NSGA2_SBX	13	2	3	1	1	2	2	2
NSGA2_PCX	37	3	1	8	8	8	1	8
GDE3	31	1	7	4	4	6	4	5
DEMOwSA	47	4	8	7	7	7	7	7
MOSaDE	30	8	4	3	3	3	6	3
MO_DE	38	5	6	5	5	5	8	4
MO_PSO	33	6	2	6	6	4	3	6
MTS	23	7	5	2	2	1	5	1

FES=50000

	Total	1.OKA2	2.SYMPART	3.S_ZDT1	4.S_ZDT2	5.S_ZDT4	6.R_ZDT4	7.S_ZDT6
NSGA2_SBX	20	2	7	2	1	1	4	3
NSGA2_PCX	31	3	3	6	4	5	3	7
GDE3	12	1	1	1	2	4	1	2
DEMOwSA	31	5	2	3	5	7	5	4
MOSaDE	37	8	6	7	6	3	2	5
MO_DE	32	4	4	4	7	6	6	1
MO_PSO	51	7	5	8	8	8	7	8
MTS	38	6	8	5	3	2	8	6

FES=500000

	Total	1.OKA2	2.SYMPART	3.S_ZDT1	4.S_ZDT2	5.S_ZDT4	6.R_ZDT4	7.S_ZDT6
NSGA2_SBX	26	2	6	4	1	2	6	5
NSGA2_PCX	32	3	7	5	2	4	4	7
GDE3	17	1	2	2	4	3	3	2
DEMOwSA	28	6	4	1	5	7	2	3
MOSaDE	31	8	5	8	7	1	1	1
MO_DE	31	4	3	3	6	6	5	4
MO_PSO	46	7	1	7	8	8	7	8
MTS	41	5	8	6	3	5	8	6
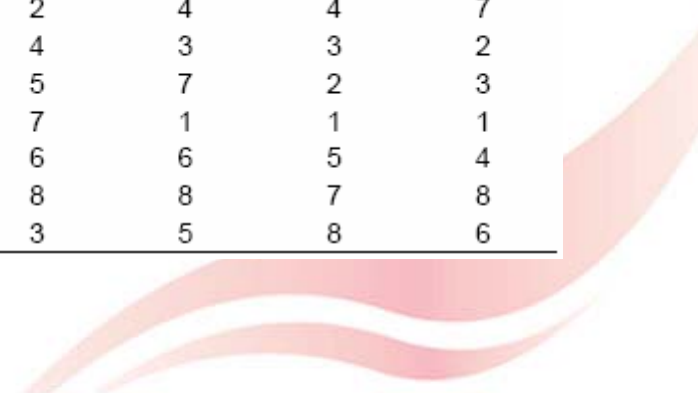
M=3, Rank(R indicator)

FES=5000

	Total	8. S_DTLZ2	9. R_DTLZ2	10. S_DTLZ3	11. WFG1	12. WFG8	13. WFG9
NSGA2_SBX	19	1	4	1	4	7	2
NSGA2_PCX	24	8	5	3	2	3	3
GDE3	25	5	3	5	6	2	4
DEMOwSA	17	4	8	2	1	1	1
MOSaDE	40	2	7	7	8	8	8
MO_DE	30	3	6	6	5	4	6
MO_PSO	32	6	1	8	7	5	5
MTS	29	7	2	4	3	6	7

FES=50000

	Total	8. S_DTLZ2	9. R_DTLZ2	10. S_DTLZ3	11. WFG1	12. WFG8	13. WFG9
NSGA2_SBX	18	5	2	1	3	6	1
NSGA2_PCX	28	7	3	5	5	3	5
GDE3	13	3	1	2	4	1	2
DEMOwSA	21	2	8	4	2	2	3
MOSaDE	37	1	7	6	7	8	8
MO_DE	31	4	6	7	6	4	4
MO_PSO	38	6	5	8	8	5	6
MTS	30	8	4	3	1	7	7

FES=500000

	Total	8. S_DTLZ2	9. R_DTLZ2	10. S_DTLZ3	11. WFG1	12. WFG8	13. WFG9
NSGA2_SBX	18	5	1	1	2	6	3
NSGA2_PCX	29	7	3	6	6	3	4
GDE3	13	2	2	2	1	1	5
DEMOwSA	27	3	8	4	4	2	6
MOSaDE	28	1	5	3	3	8	8
MO_DE	29	4	6	7	7	4	1
MO_PSO	36	6	7	8	8	5	2
MTS	36	8	4	5	5	7	7

M=3, Rank(Hypervolumn)

FES=5000

	Total	8. S_DTLZ2	9. R_DTLZ2	10. S_DTLZ3	11. WFG1	12. WFG8	13. WFG9
NSGA2_SBX	17	1	3	2	8	1	2
NSGA2_PCX	28	8	5	5	2	4	4
GDE3	22	4	4	3	5	3	3
DEMOwSA	20	6	6	4	1	2	1
MOSaDE	42	5	8	6	7	8	8
MO_DE	31	2	7	7	4	5	6
MO_PSO	30	3	2	8	6	6	5
MTS	26	7	1	1	3	7	7

FES=50000

	Total	8. S_DTLZ2	9. R_DTLZ2	10. S_DTLZ3	11. WFG1	12. WFG8	13. WFG9
NSGA2_SBX	21	6	2	1	8	1	3
NSGA2_PCX	29	7	3	5	4	4	6
GDE3	12	3	1	2	3	2	1
DEMOwSA	22	2	8	6	1	3	2
MOSaDE	35	1	7	4	7	8	8
MO_DE	32	4	6	7	5	5	5
MO_PSO	34	5	5	8	6	6	4
MTS	31	8	4	3	2	7	7

FES=500000

	Total	8. S_DTLZ2	9. R_DTLZ2	10. S_DTLZ3	11. WFG1	12. WFG8	13. WFG9
NSGA2_SBX	23	6	1	2	8	1	5
NSGA2_PCX	30	7	3	5	5	4	6
GDE3	12	2	2	3	1	2	2
DEMOwSA	25	3	8	4	3	3	4
MOSaDE	25	1	5	1	2	8	8
MO_DE	29	4	6	7	6	5	1
MO_PSO	36	5	7	8	7	6	3
MTS	36	8	4	6	4	7	7

M=5, Rank(R indicator)

FES=5000

	Total	8. S_DTLZ2	9. R_DTLZ2	10. S_DTLZ3	11. WFG1	12. WFG8	13. WFG9
NSGA2_SBX	15	1	1	1	8	2	2
NSGA2_PCX	16	4	3	2	3	3	1
GDE3	27	5	5	4	5	4	4
DEMOwSA	36	8	8	5	2	6	7
MOSaDE	37	3	7	6	7	8	6
MO_DE	33	6	6	7	4	5	5
MO_PSO	40	7	4	8	6	7	8
MTS	12	2	2	3	1	1	3

FES=50000

	Total	8. S_DTLZ2	9. R_DTLZ2	10. S_DTLZ3	11. WFG1	12. WFG8	13. WFG9
NSGA2_SBX	11	1	1	1	6	1	1
NSGA2_PCX	31	8	4	8	5	4	2
GDE3	17	3	3	3	2	2	4
DEMOwSA	29	2	8	4	4	6	5
MOSaDE	39	4	6	5	8	8	8
MO_DE	28	6	7	6	3	3	3
MO_PSO	39	7	5	7	7	7	6
MTS	22	5	2	2	1	5	7

FES=500000

	Total	8. S_DTLZ2	9. R_DTLZ2	10. S_DTLZ3	11. WFG1	12. WFG8	13. WFG9
NSGA2_SBX	9	1	1	1	4	1	1
NSGA2_PCX	33	7	5	8	5	6	2
GDE3	16	4	3	2	1	2	4
DEMOwSA	30	2	8	4	7	4	5
MOSaDE	31	3	4	5	3	8	8
MO_DE	30	6	6	6	6	3	3
MO_PSO	41	8	7	7	8	5	6
MTS	26	5	2	3	2	7	7

M=5, Rank(Hypervolumn)

FES=5000

	Total	8. S_DTLZ2	9. R_DTLZ2	10. S_DTLZ3	11. WFG1	12. WFG8	13. WFG9
NSGA2_SBX	14	1	1	1	8	1	2
NSGA2_PCX	18	7	3	2	3	2	1
GDE3	22	2	5	4	5	3	3
DEMOwSA	32	8	8	6	1	4	5
MOSaDE	37	3	7	5	7	8	7
MO_DE	30	4	6	7	4	5	4
MO_PSO	33	5	2	8	6	6	6
MTS	30	6	4	3	2	7	8

FES=50000

	Total	8. S_DTLZ2	9. R_DTLZ2	10. S_DTLZ3	11. WFG1	12. WFG8	13. WFG9
NSGA2_SBX	12	2	1	1	6	1	1
NSGA2_PCX	31	8	5	8	5	3	2
GDE3	15	3	2	2	3	2	3
DEMOwSA	28	4	8	4	4	4	4
MOSaDE	35	1	6	5	8	8	7
MO_DE	31	6	7	6	2	5	5
MO_PSO	35	5	4	7	7	6	6
MTS	29	7	3	3	1	7	8

FES=500000

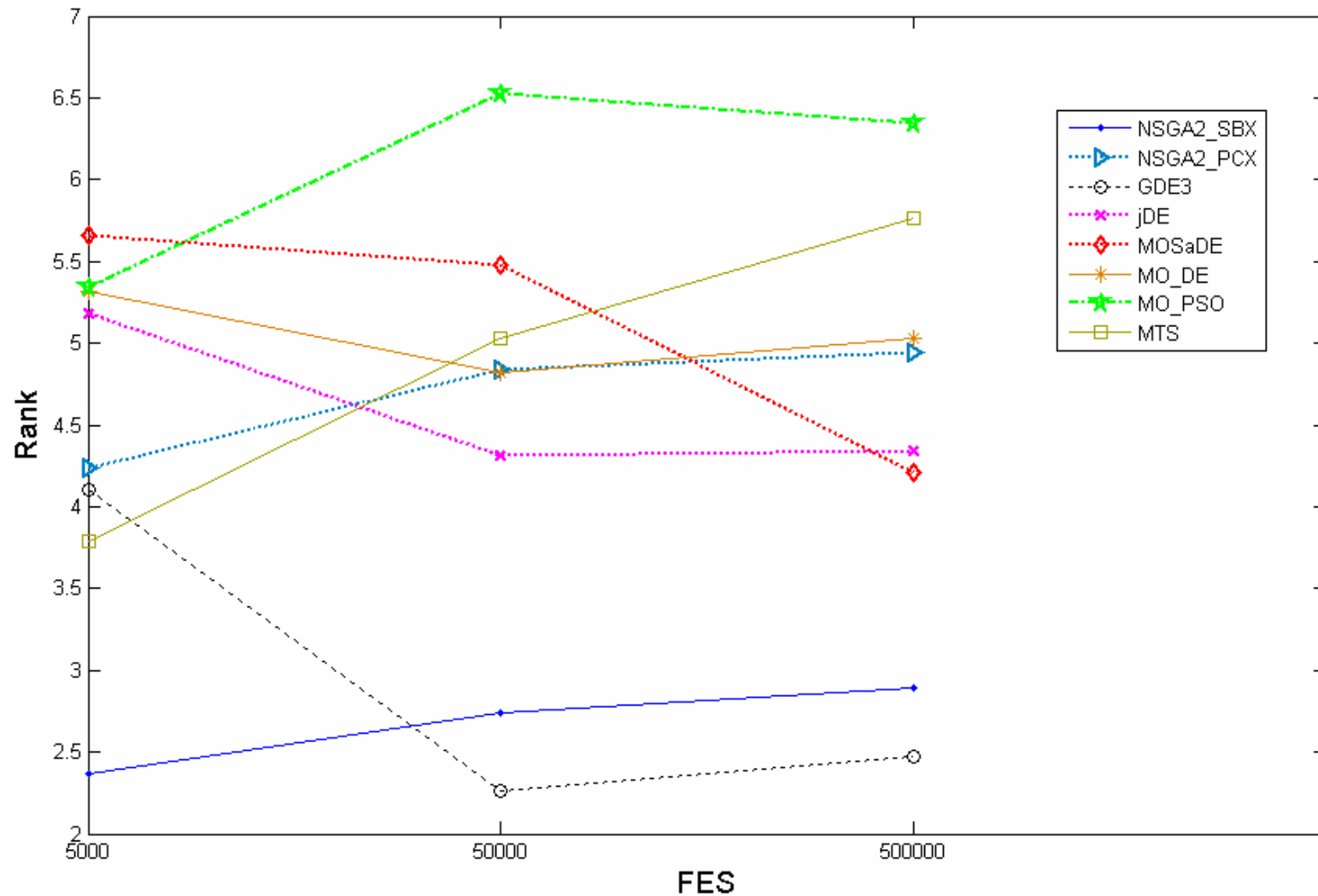|	Total	8. S_DTLZ2	9. R_DTLZ2	10. S_DTLZ3	11. WFG1	12. WFG8	13. WFG9
NSGA2_SBX	10	3	1	1	3	1	1
NSGA2_PCX	33	6	5	8	5	3	6
GDE3	14	4	2	2	1	2	3
DEMOwSA	26	2	8	3	7	4	2
MOSaDE	26	1	4	4	2	7	8
MO_DE	37	7	7	7	6	6	4
MO_PSO	35	5	6	6	8	5	5
MTS	35	8	3	5	4	8	7

Summarized Results - Rank by I_{R2}
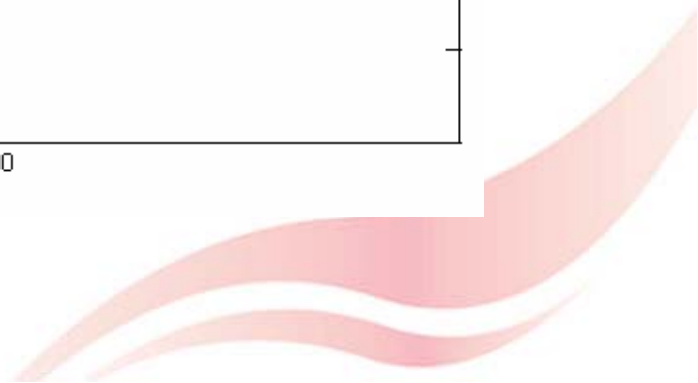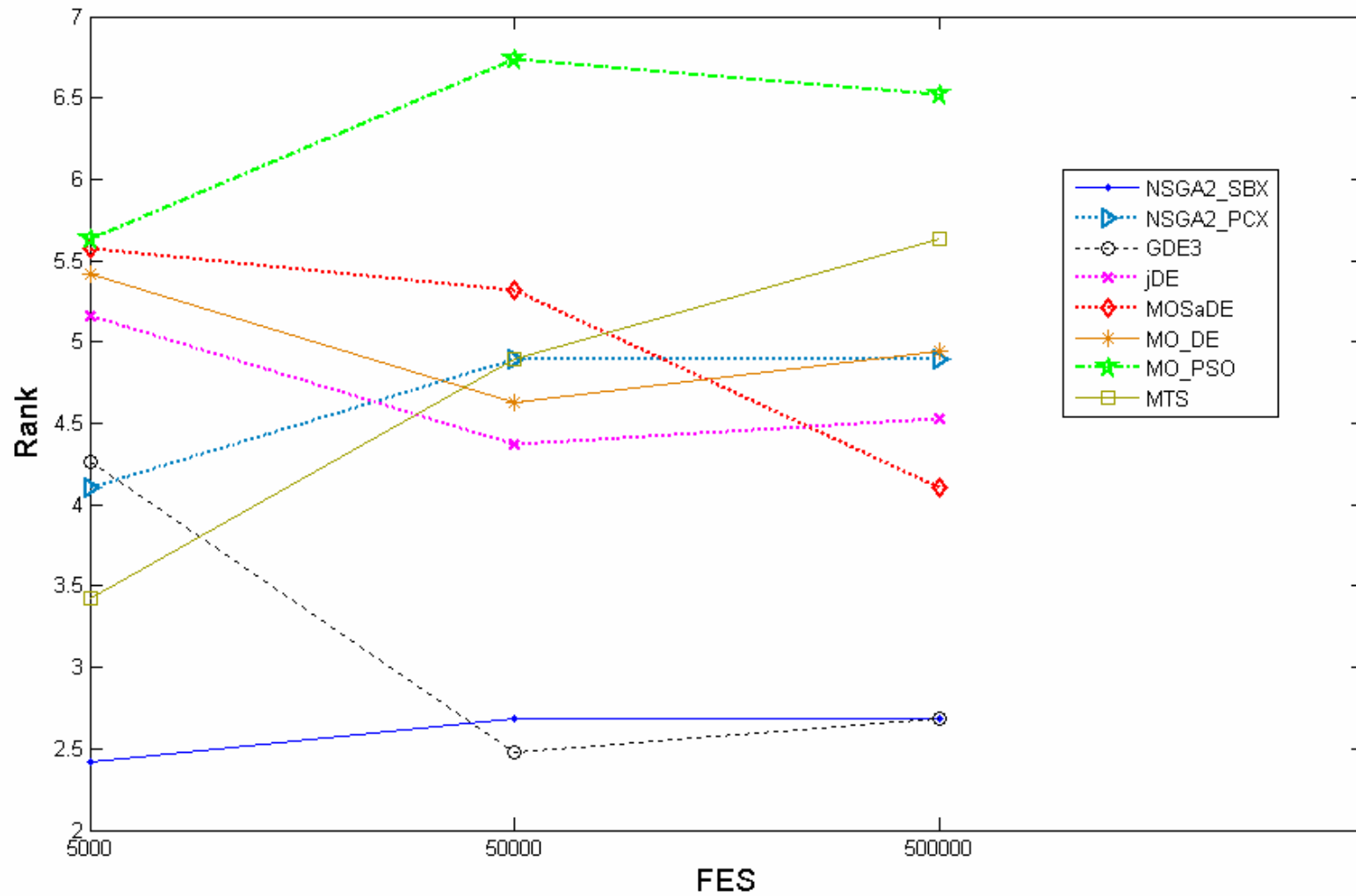
Summarized Results - Rank by $I_{\bar{H}}$
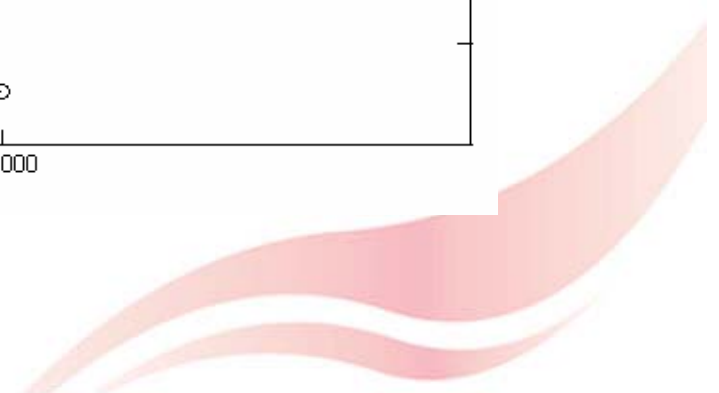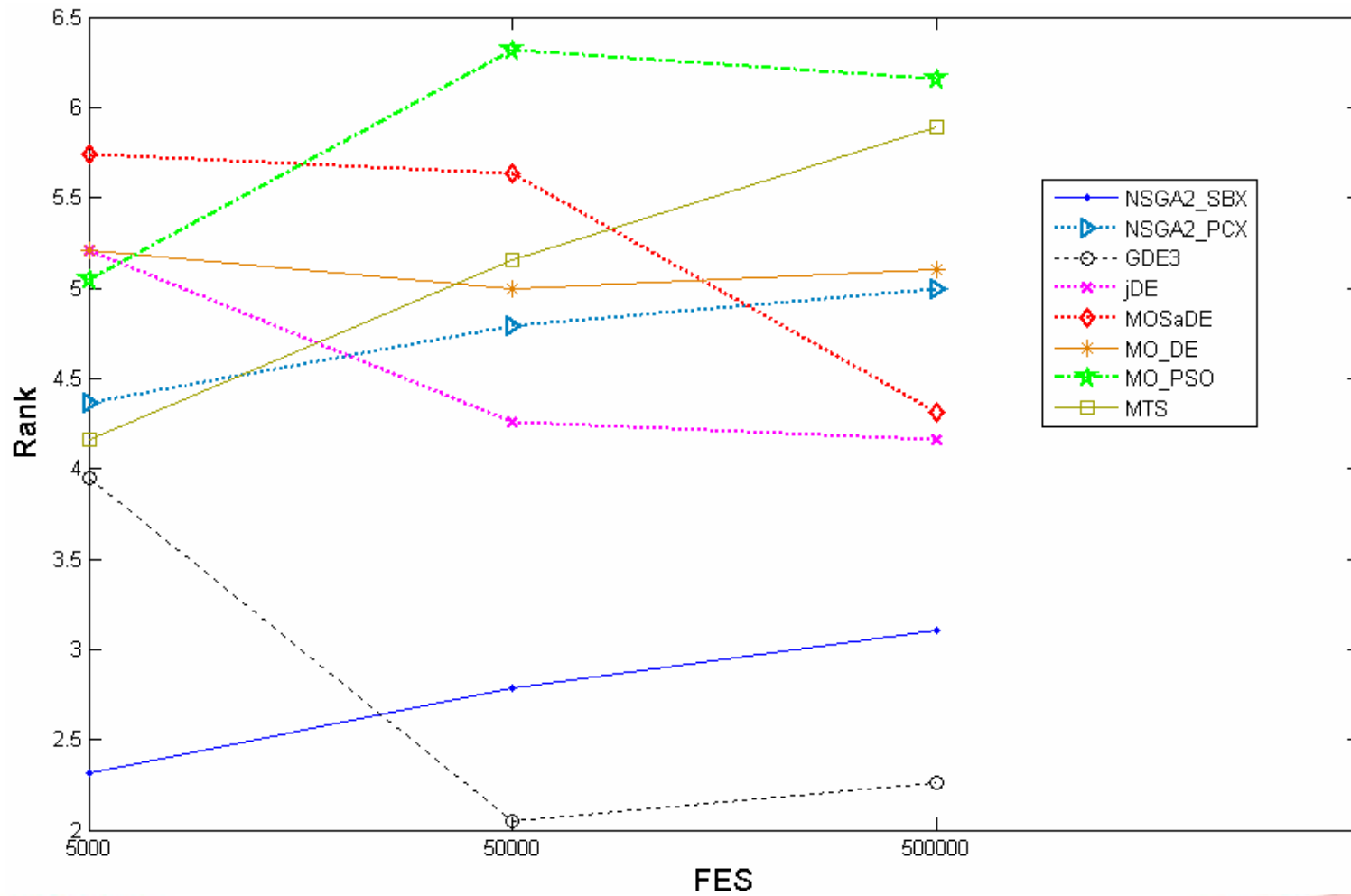
Summarized Results - Rank by I_{R2} and $I_{\bar{H}}$

Rank (I_{R2} and $I_{\bar{H}}$) on all test problems

FES=5000

	R	H	Total
NSGA2_SBX	2.42(1)	2.32(1)	**4.74(1)**
NSGA2_PCX	4.11(3)	4.37(4)	8.47(4)
GDE3	4.26(4)	3.95(2)	8.21(3)
DEMOwSA	5.16(5)	5.21(6.5)	10.37(5)
MOSaDE	5.58(7)	5.74(8)	11.32(8)
MO_DE	5.42(6)	5.21(6.5)	10.63(6)
MO_PSO	5.63(8)	5.05(5)	10.68(7)
MTS	3.42(2)	4.16(3)	7.58(2)

FES=50000

	R	H	Total
NSGA2_SBX	2.68(2)	2.79(2)	5.47(2)
NSGA2_PCX	4.89(5.5)	4.79(4)	9.68(5)
GDE3	2.47(1)	2.05(1)	**4.53(1)**
DEMOwSA	4.37(3)	4.26(3)	8.63(3)
MOSaDE	5.32(7)	5.63(7)	10.95(7)
MO_DE	4.63(4)	5.00(5)	9.63(4)
MO_PSO	6.74(8)	6.32(8)	13.05(8)
MTS	4.89(5.5)	5.16(6)	10.05(6)

FES=500000

	R	H	Total
NSGA2_SBX	2.68(1.5)	3.11(2)	5.79(2)
NSGA2_PCX	4.89(5)	5.00(5)	9.89(5)
GDE3	2.68(1.5)	2.26(1)	**4.95(1)**
DEMOwSA	4.53(4)	4.16(3)	8.68(4)
MOSaDE	4.11(3)	4.32(4)	8.42(3)
MO_DE	4.95(6)	5.11(6)	10.05(6)
MO_PSO	6.53(8)	6.16(8)	12.68(8)
MTS	5.63(7)	5.89(7)	11.53(7)

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Rank on all test problems of FES=5000,50000,500000 together

	R	H
NSGA2_SBX	7.79(1)	8.21(1)
NSGA2_PCX	13.89(3)	14.16(4)
GDE3	9.42(2)	8.26(2)
DEMOwSA	14.05(5)	13.63(3)
MOSaDE	15.00(6.5)	15.68(7)
MO_DE	15.00(6.5)	15.32(6)
MO_PSO	18.89(8)	17.53(8)
MTS	13.95(4)	15.21(5)