Benchmark Functions for CEC 2015 Special Session and Competition on Dynamic Multi-objective Optimization

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I. Introduction

Most real-world optimization problems have more than one objective, with at least two objectives that are in conflict with one another. The conflicting objectives of the optimization problem lead to an optimization problem where a single solution does not exist, as is the case with single-objective optimisation problems (SOOPs). In stead of a single solution, a set of optimal trade-off solutions exists, referred to as the Pareto-optimal front (POF) or Pareto front. This kind of optimization problems are referred to as multi-objective optimisation problems (MOOPs).

In many real-world situations the environment does not remain static, but is dynamic and changes over time. However, in recent years most research was focussed on either static MOOPs (SMOOPs) [1], [2], [3], [4], [5], [6] or dynamic single-objective optimisation problems (DSOOPs) [7], [8], [9], [10], [11], [12], [13]. When solving dynamic multi-objective optimisation problems (DMOOPs) an algorithm has to track the changing POF over time, while finding solutions as close as possible to the true POF and maintaining a diverse set of solutions. Some of the major challenges in the field of dynamic multi-objective optimization (DMOO) are a lack of a standard set of benchmark functions, a lack of standard performance measures, issues with performance measures currently being used for dynamic multi-objective optimisation (DMOO) and a lack of a comprehensive analysis of existing algorithms applied to DMOO [14].

Recently, some of these challenges were addressed: a comprehensive overview of existing DMOOPs were presented [15], characteristics of an ideal benchmark function suite were proposed and DMOOPs were suggested for each of these characteristics [15], [16]; and a comprehensive overview of performance measures currently used for DMOO [17], [18] was presented and issues with some of these measures when applied to DMOO were highlighted [17], [18].

However, what is still lacking is a comprehensive analysis of algorithms proposed for DMOO and a bechmark algorithm(s) to compare newly proposed algorithms against. Therefore, a unified framework should be adopted for evaluating DMOO algorithms in order to provide a common platform for future research in the field. In this report 12 benchmark functions with different characteristics and from various DMOOP types (as defined by Farina *et. al* [19]) are included. Performance measures are also suggested to compare the various DMOO algorithms.

The rest of the report is outlined as follows: Section II presents the mathematical formula and properties of the various benchmark functions. The experimental setup is presented in Section III. Section IV discusses the process that is followed to rank the entries of this competition. The process that participants should follow to enter the competition is discussed in Section V.

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II. Benchmark Functions

This section presents the benchmarks functions for this competition. Section II-A presents a summary of the benchmark function set. The mathematical formula and properties of the benchmark functions are discussed in more detail in Section II-B. It should be noted that the values of the benchmark function parameters for the competition are presented in Section III.

A. Summary of Benchmark Functions

The benchmark set includes the following 12 DMOOPs:

- FDA4 [19]
- FDA5 [19]
- FDA5$_{iso}$ [15], [16]
- FDA5$_{dec}$ [15], [16]
- DIMP2 modified from [20]
- dMOP2 modified from [21]
- dMOP2$_{iso}$ modified from [15], [16]
- dMOP2$_{dec}$ modified from [15], [16]
- dMOP3 modified from [21]
- HE2 [22]
- HE7 [15], [16]
- HE9 [15], [16]

The benchmark functions have the following properties:

- All functions, except FDA4 and FDA5, are 2-objective functions.
- FDA4 and FDA5 are 3-objective functions.
- Type I DMOOPs: FDA4, DIMP2, dMOP3
- Type II DMOOPs: FDA5, FDA5$_{iso}$, FDA5$_{dec}$, dMOP2, dMOP2$_{iso}$, dMOP2$_{dec}$
- Type III DMOOPs: HE2, HE7, HE9
- POF' spread of solutions changes over time for dMOP3, FDA5, FDA5$_{iso}$ and FDA5$_{dec}$

- The POF changes from convex to concave and vice versa for dMOP2, dMOP2$_{iso}$, dMOP2$_{dec}$, HE7 and HE9
- The POS of HE7 and HE9 is complex, i.e. a non-linear function
- The POF of HE2 is discontinuous
- The POF of dMOP3 and DIMP2 is convex
- Each decision variable of DIMP2 has its own rate of change

B. Definitions and Properties of Benchmark Functions

This section presents the mathematical formula and properties of the benchmark functions.

FDA4

$$\text{FDA4} = \begin{cases} Minimize: \mathbf{f}(\mathbf{x},t) = (f_1(\mathbf{x}, g(\mathbf{x}_{\text{II}},t)), \ldots, f_k(\mathbf{x}, g(\mathbf{x}_{\text{II}},t))) \\ f_1(\mathbf{x},g,t) = (1+g(\mathbf{x}_{\text{II}},t)) \prod_{i=1}^{M-1} \cos\left(\frac{x_i \pi}{2}\right) \\ f_k(\mathbf{x},g,t) = (1+g(\mathbf{x}_{\text{II}},t)) \left(\prod_{i=1}^{M-k} \cos\left(\frac{x_i \pi}{2}\right)\right) \\ \qquad \sin\left(\frac{x_{M-k+1}\pi}{2}\right), \forall k = 2, \ldots, M-1 \\ f_M(\mathbf{x},g,t) = (1+g(\mathbf{x}_{\text{II}},t)) \sin\left(\frac{x_1 \pi}{2}\right) \\ where: \\ g(\mathbf{x}_{\text{II}},t) = \sum_{x_i \in \mathbf{x}_{II}} (x_i - G(t))^2, \quad G(t) = |sin(0.5\pi t)| \\ t = \frac{1}{n_t} \left\lfloor \frac{\tau}{\tau_t} \right\rfloor \\ \mathbf{x}_{\text{II}} = (x_M, \ldots, x_n); \quad x_i \in [0,1], \ \forall i = 1, \ldots, n \end{cases} \tag{1}$$

Properties of FDA4

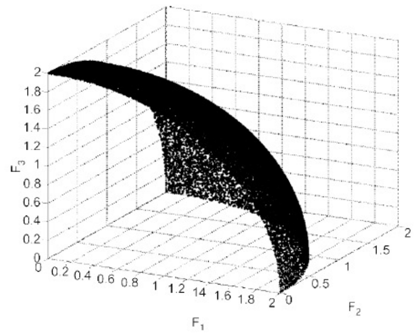
- Set $k = 2$ and $M = 3$
- 3-objective function
- Type I DMOOP
- Non-convex POF - spherical surface
- POF is $f_1^2 + f_2^2 + f_3^2 = 1$ (refer to Figure 1(a))
- Pareto-optimal set (POS) is $x_i = G(t), \ \forall x_i \in \mathbf{x_{II}}$

FDA5

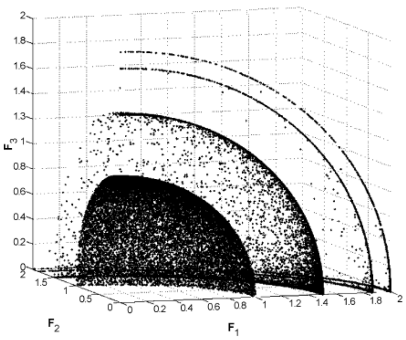
$$\text{FDA5} = \begin{cases} Minimize: \mathbf{f}(\mathbf{x},t) = (f_1(\mathbf{x}, g(\mathbf{x}_{\text{II}},t)), \ldots, f_k(\mathbf{x}, g(\mathbf{x}_{\text{II}},t))) \\ f_1(\mathbf{x},g,t) = (1+g(\mathbf{x}_{\text{II}},t)) \prod_{i=1}^{M-1} \cos\left(\frac{y_i \pi}{2}\right) \\ f_k(\mathbf{x},g,t) = (1+g(\mathbf{x}_{\text{II}},t)) \left(\prod_{i=1}^{M-k} \cos\left(\frac{y_i \pi}{2}\right)\right) \\ \qquad \sin\left(\frac{y_{M-k+1}\pi}{2}\right), \forall k = 2, \ldots, M-1 \\ f_M(\mathbf{x},g,t) = (1+g(\mathbf{x}_{\text{II}},t)) \sin\left(\frac{y_1 \pi}{2}\right) \\ where: \\ g(\mathbf{x}_{\text{II}},t) = G(t) + \sum_{x_i \in \mathbf{x}_{II}} (x_i - G(t))^2 \\ G(t) = |sin(0.5\pi t)|, \quad t = \frac{1}{n_t} \left\lfloor \frac{\tau}{\tau_t} \right\rfloor \\ y_i = x_i^{F(t)}, \quad \forall i = 1, \ldots, (M-1) \\ F(t) = 1 + 100 \sin^4(0.5\pi t) \\ \mathbf{x}_{\text{II}} = (x_M, \ldots, x_n); \quad x_i \in [0,1], \ \forall i = 1, \ldots, n \end{cases} \tag{2}$$

Properties of FDA5

- Set $M = 3$
- 3-objective function
- Type II DMOOP
- Non-convex POF
- Spread of solutions in the POF changes over time
- POF is $f_1^2 + f_2^2 + f_3^2 = (1 + G(t))^2$ (refer to Figure 1(b))
- POS is $x_i = G(t), \ \forall x_i \in \mathbf{x}_{\text{II}}$



(a) POF of FDA4[19]



(b) POF of FDA5 for four time steps[19]

Fig. 1. POF of FDA4 and FDA5 for three objective functions. The size of the sphere's radius of FDA5's POF changes in a cyclic manner as the value of G changes over time. The radius increases over time and then decreasing to the value of 1.0

FDA5$_{iso}$

$$\text{FDA5}_{iso} = \begin{cases} Minimize: \mathbf{f}(\mathbf{x},t) = (f_1(\mathbf{x}, g(\mathbf{x_{II}},t)), \ldots, \\ \qquad f_k(\mathbf{x}, g(\mathbf{x_{II}},t))) \\ f_1(\mathbf{x},g,t) = (1+g(\mathbf{x_{II}},t)) \prod_{i=1}^{M-1} \cos\left(\frac{y_i \pi}{2}\right) \\ f_k(\mathbf{x},g,t) = (1+g(\mathbf{x_{II}},t)) \left(\prod_{i=1}^{M-k} \cos\left(\frac{y_i \pi}{2}\right)\right) \\ \qquad \sin\left(\frac{y_{M-k+1}\pi}{2}\right), \forall k = 2, \ldots, M-1 \\ f_M(\mathbf{x},g,t) = (1+g(\mathbf{x_{II}},t)) \sin\left(\frac{y_1 \pi}{2}\right) \\ where: \\ g(\mathbf{x_{II}},t) = G(t) + \sum_{x_i \in \mathbf{x_{II}}} (y_j - G(t))^2 \\ G(t) = |sin(0.5\pi t)|, \quad t = \frac{1}{n_t} \left\lfloor \frac{\tau}{\tau_t} \right\rfloor \\ F(t) = 1 + 100 \sin^4(0.5\pi t) \\ y_i = x_i^{F(t)}, \quad \forall i = 1, \ldots, (M-1); \\ y_j = y^*(x_i, A, B, C), \quad \forall x_i \in \mathbf{x_{II}} \\ y^*(x_i, A, B, C) = A + \min(0, \lfloor x_i - B \rfloor) \frac{A(B - x_i)}{B} - \\ \qquad \min(0, \lfloor C - x_i \rfloor) \frac{(1-A)(x_i - C)}{1-C} \\ \mathbf{x_{II}} = (x_M, \ldots, x_n), \quad x_i \in [0,1], \ \forall i = 1, \ldots, n \end{cases} \tag{3}$$

Properties of FDA5$_{iso}$

- Set $A = G(t)$, $B = 0.001$ and $C = 0.05$
- Set $M = 3$
- Type II DMOOP
- Isolated, non-convex POF
- Spread of solutions in the POF changes over time
- POF is $f_1^2 + f_2^2 + f_3^2 = (1 + G(t))^2$ (refer to Figure 1(b))
- POS is $y_j = G(t), \ \forall x_i \in \mathbf{x_{II}}$

FDA5$_{dec}$

$$\text{FDA5}_{dec} = \begin{cases} Minimize: \mathbf{f}(\mathbf{x},t) = (f_1(\mathbf{x}, g(\mathbf{x_{II}},t)), \dots, \\ \quad f_k(\mathbf{x}, g(\mathbf{x_{II}},t))) \\ f_1(\mathbf{x},g,t) = (1+g(\mathbf{x_{II}},t)) \prod_{i=1}^{M-1} \cos\left(\frac{y_i \pi}{2}\right) \\ f_k(\mathbf{x},g,t) = (1+g(\mathbf{x_{II}},t)) \left(\prod_{i=1}^{M-k} \cos\left(\frac{y_i \pi}{2}\right)\right) \\ \quad \sin\left(\frac{y_{M-k+1}\pi}{2}\right), \forall k = 2, \dots, M-1 \\ f_M(\mathbf{x},g,t) = (1+g(\mathbf{x_{II}},t)) \sin\left(\frac{y_1 \pi}{2}\right) \\ where: \\ g(\mathbf{x_{II}},t) = G(t) + \sum_{x_i \in \mathbf{x_{II}}} (y_j - G(t))^2 \\ G(t) = |sin(0.5\pi t)|, \quad t = \frac{1}{n_t}\left\lfloor \frac{\tau}{\tau_t} \right\rfloor \\ F(t) = 1 + 100 \sin^4(0.5\pi t) \\ y_i = x_i^{F(t)}, \quad \forall i = 1, \dots, (M-1); \\ y_j = y^*(x_i, A, B, C), \quad \forall x_i \in \mathbf{x_{II}} \\ y^*(x_i, A, B, C) = \left(\frac{\lfloor x_i - A + B \rfloor \left(1 - C + \frac{A-B}{B}\right)}{A-B} + \frac{1}{B} + \right. \\ \quad \left. \frac{\lfloor A + B - x_i \rfloor \left(1 - C + \frac{1-A-B}{B}\right)}{1-A-B} \right) (|x_i - A| - B) + 1 \\ \mathbf{x_{II}} = (x_M, \dots, x_n), \quad x_i \in [0,1], \ \forall i = 1, \dots, n \end{cases} \quad (4)$$

Properties of FDA5$_{dec}$

- Set $A = G(t)$, $B = 0.001$ and $C = 0.05$
- Set $M = 3$
- Type II DMOOP
- Deceptive, non-convex POF
- POF is $f_1^2 + f_2^2 + f_3^2 = (1 + G(t))^2$ (refer to Figure 1(b))
- POS is $y_j = G(t), \ \forall x_i \in \mathbf{x_{II}}$

DIMP2

$$\text{DIMP2} = \begin{cases} Minimize: f(\mathbf{x},t) = (f_1(\mathbf{x_I}), g(\mathbf{x_{II}},t) \cdot h(f_1(\mathbf{x_I}), \\ \quad g(\mathbf{x_{II}},t))) \\ f_1(\mathbf{x_I}) = x_1 \\ g(\mathbf{x_{II}},t) = 1 + 2(n-1) + \sum_{x_i \in \mathbf{x_{II}}} [(x_i - G_i(t))^2 - \\ \quad 2\cos(3\pi(x_i - G_i(t)))] \\ h(f_1, g) = 1 - \sqrt{\frac{f_1}{g}} \\ where: \\ G_i(t) = \sin\left(0.5\pi t + 2\pi\left(\frac{i}{n+1}\right)\right)^2, \quad t = \frac{1}{n_t}\left\lfloor \frac{\tau}{\tau_t} \right\rfloor \\ \mathbf{x_I} \in [0,1]; \quad \mathbf{x_{II}} \in [-2,2]^{n-1} \end{cases} \quad (5)$$

Properties of DIMP2

- Type I DMOOP
- Convex POF
- Each decision variable has its own rate of change
- POF is $1 - \sqrt{f_1}$ (refer to Figure 2)
- POS is $x_i = G(t), \ \forall x_i \in \mathbf{x_{II}}$

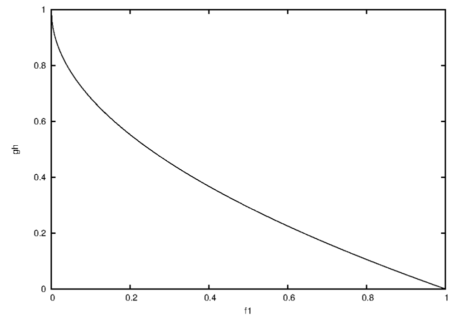


Fig. 2. POF of DIMP2 with $n_t = 10$ and $\tau_t = 10$ for 1000 iterations [19].

dMOP2

$$\text{dMOP2} = \begin{cases} Minimize: f(\mathbf{x},t) = (f_1(\mathbf{x_I}), g(\mathbf{x_{II}},t) \cdot h(f_1(\mathbf{x_I}), \\ \quad g(\mathbf{x_{II}},t),t)) \\ f_1(\mathbf{x_I}) = x_1 \\ g(\mathbf{x_{II}},t) = 1 + 9\sum_{x_i \in \mathbf{x_{II}}} (x_i - G(t))^2 \\ h(f_1, g, t) = 1 - \left(\frac{f_1}{g}\right)^{H(t)} \\ where: \\ H(t) = 0.75 \sin(0.5\pi t) + 1.25, \\ G(t) = \sin(0.5\pi t) \\ t = \frac{1}{n_t}\left\lfloor \frac{\tau}{\tau_t} \right\rfloor \\ x_i \in [0,1]; \quad \mathbf{x_I} = (x_1) \\ \mathbf{x_{II}} = (x_2, \dots, x_n) \end{cases} \quad (6)$$

Properties of dMOP2

- Type II DMOOP
- POF changes from convex to concave, and vice versa
- POF is $1 - f_1^{H(t)}$ (refer to Figure 3)
- POS is $x_i = G(t), \ \forall x_i \in \mathbf{x_{II}}$

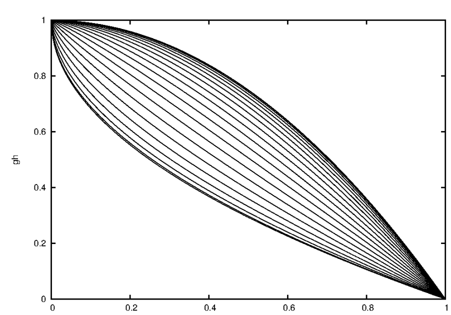


Fig. 3. POF of dMOP2 with $n_t = 10$ and $\tau_t = 10$ for 1000 iterations. POF changes in a cyclic manner over time, by moving either from the middle line to the top line for certain time steps or from the bottom line to the middle line for the other time steps.

dMOP2$_{iso}$

$$\text{dMOP2}_{iso} = \begin{cases} Minimize: f(\mathbf{x},t) = (f_1(\mathbf{x_I}), g(\mathbf{x_{II}},t)\cdot h(f_1(\mathbf{x_I}), \\ \quad g(\mathbf{x_{II}},t),t)) \\ f_1(\mathbf{x_I}) = x_1 \\ g(\mathbf{x_{II}},t) = 1 + 9\sum_{x_i\in\mathbf{x_{II}}}(y_i - G(t))^2 \\ h(f_1,g,t) = 1 - \left(\frac{f_1}{g}\right)^{H(t)} \\ where: \\ y_i = y^*(x_i,A,B,C), \quad \forall x_i \in \mathbf{x_{II}} \\ y^*(x_i,A,B,C) = A + \min(0, \lfloor x_i - B\rfloor)\frac{A(B-x_i)}{B} - \\ \quad \min(0,\lfloor C - x_i\rfloor)\frac{(1-A)(x_i-C)}{1-C} \\ H(t) = 0.75\sin(0.5\pi t) + 1.25 \\ G(t) = \sin(0.5\pi t), \quad t = \frac{1}{n_t}\left\lfloor\frac{\tau}{\tau_t}\right\rfloor \\ x_i \in [0,1]; \quad \mathbf{x_I} = (x_1), \quad \mathbf{x_{II}} = (x_2,\dots,x_n) \end{cases} \tag{7}$$

Properties of dMOP2$_{iso}$

- Set $A = G(t)$, $B = 0.001$ and $C = 0.05$
- Type II DMOOP
- Isolated POF
- POF changes from convex to concave, and vice versa
- POF is $1 - f_1^{H(t)}$ (refer to Figure 3)
- POS is $y_i = G(t),\ \forall x_i \in \mathbf{x_{II}}$

dMOP2$_{dec}$

$$\text{dMOP2}_{dec} = \begin{cases} Minimize: f(\mathbf{x},t) = (f_1(\mathbf{x_I}), g(\mathbf{x_{II}},t)\cdot h(f_1(\mathbf{x_I}), \\ \quad g(\mathbf{x_{II}},t),t)) \\ f_1(\mathbf{x_I}) = x_1 \\ g(\mathbf{x_{II}},t) = 1 + 9\sum_{x_i\in\mathbf{x_{II}}}(y_i - G(t))^2 \\ h(f_1,g,t) = 1 - \left(\frac{f_1}{g}\right)^{H(t)} \\ where: \\ y_i = y^*(x_i,A,B,C), \quad \forall x_i \in \mathbf{x_{II}} \\ y^*(x_i,A,B,C) = \Bigg(\frac{\lfloor x_i - A + B\rfloor\left(1 - C + \frac{A-B}{B}\right)}{A-B} + \frac{1}{B} + \\ \quad \frac{\lfloor A + B - x_i\rfloor\left(1 - C + \frac{1-A-B}{B}\right)}{1-A-B}\Bigg)(|x_i - A| - B) + 1 \\ H(t) = 0.75\sin(0.5\pi t) + 1.25 \\ G(t) = \sin(0.5\pi t), \quad t = \frac{1}{n_t}\left\lfloor\frac{\tau}{\tau_t}\right\rfloor \\ x_i \in [0,1]; \quad \mathbf{x_I} = (x_1), \quad \mathbf{x_{II}} = (x_2,\dots,x_n) \end{cases} \tag{8}$$

Properties of dMOP2$_{dec}$

- Set $A = G(t)$, $B = 0.001$ and $C = 0.05$
- Type II DMOOP
- Deceptive POF
- POF changes from convex to concave, and vice versa
- POF is $1 - f_1^{H(t)}$ (refer to Figure 3)
- POS is $y_i = G(t),\ \forall x_i \in \mathbf{x_{II}}$

dMOP3

$$\text{dMOP3} = \begin{cases} Minimize: f(\mathbf{x},t) = (f_1(\mathbf{x_I}), g(\mathbf{x_{II}},t)\cdot h(f_1(\mathbf{x_I}), \\ \quad g(\mathbf{x_{II}},t))) \\ f_1(\mathbf{x_I}) = x_r \\ g(\mathbf{x_{II}},t) = 1 + 9\sum_{x_i\in\mathbf{x_{II}}\setminus x_r}(x_i - G(t))^2 \\ h(f_1,g) = 1 - \sqrt{\frac{f_1}{g}} \\ where: \\ G(t) = \sin(0.5\pi t), \quad t = \frac{1}{n_t}\left\lfloor\frac{\tau}{\tau_t}\right\rfloor \\ x_i \in [0,1]; \quad r = \bigcup(1,2,\dots,n) \end{cases} \tag{9}$$

Properties of dMOP3

- Type I DMOOP
- Convex POF
- Spread of the POF solutions changes over time
- POF is $1 - \sqrt{f_1}$ (refer to Figure 2) POS is $x_i = G(t),\ \forall x_i \in \mathbf{x_{II}}$

HE2

$$\text{HE2} = \begin{cases} Minimize: f(\mathbf{x},t) = (f_1(\mathbf{x_I}), g(\mathbf{x_{II}})\cdot h(f_1(\mathbf{x_I}), \\ \quad g(\mathbf{x_{II}}),t)) \\ f_1(\mathbf{x_I}) = x_i \\ g(\mathbf{x_{II}}) = 1 + \frac{9}{n-1}\sum_{x_i\in\mathbf{x_{II}}} x_i \\ h(f_1,g,t) = 1 - \left(\sqrt{\frac{f_1}{g}}\right)^{H(t)} - \left(\frac{f_1}{g}\right)^{H(t)}\sin(10\pi f_1) \\ where: \\ H(t) = 0.75\sin(0.5\pi t) + 1.25, \quad t = \frac{1}{n_t}\left\lfloor\frac{\tau}{\tau_t}\right\rfloor \\ x_i \in [0,1]; \quad \mathbf{x_I} = (x_1); \quad \mathbf{x_{II}} = (x_2,\dots,x_n) \end{cases} \tag{10}$$

Properties of HE2

- Type III DMOOP
- Discontinuous POF, with various disconnected continuous sub-regions
- POF is $1 - \left(\sqrt{f1}\right)^{H(t)} - f_1^{H(t)}\sin(0.5\pi f_1)$
- POS is $x_i = 0,\ \forall x_i \in \mathbf{x_{II}}$

HE7

$$\text{HE7} = \begin{cases} Minimize: \mathbf{f}(\mathbf{x},t) = (f_1(\mathbf{x}), g(\mathbf{x},t)\cdot h(f_1(\mathbf{x}), \\ \quad g(\mathbf{x},t))) \\ f_1(\mathbf{x}) = x_1 + \frac{2}{|J_1|}\sum_{j\in J_1}\big(x_j - [0.3x_1^2\cos\big(24\pi x_1 + \frac{4j\pi}{n}\big) \\ \quad +\ 0.6x_1]\cos\big(6\pi x_1 + \frac{j\pi}{n}\big)\big)^2 \\ g(\mathbf{x}) = 2 - \sqrt{x_1} + \frac{2}{|J_2|}\sum_{j\in J_2}\big(x_j - [0.3x_1^2\cos\big(24\pi x_1 + \\ \quad \frac{4j\pi}{n}\big) + 0.6x_1]\sin\big(6\pi x_1 + \frac{j\pi}{n}\big)\big)^2 \\ h(f_1,g) = 1 - \left(\frac{f_1}{g}\right)^{H(t)} \\ where: \\ H(t) = 0.75\sin(0.5\pi t) + 1.25, \quad t = \frac{1}{n_t}\left\lfloor\frac{\tau}{\tau_t}\right\rfloor \\ J_1 = \{j|\ \text{j is odd and}\ \ 2 \le j \le n\} \\ J_2 = \{j|\ \text{j is even and}\ \ 2 \le j \le n\} \\ x_1 \in [0,1], \quad x_i \in [-1,1], \quad \forall i = 2,3,\dots,n \end{cases} \tag{11}$$

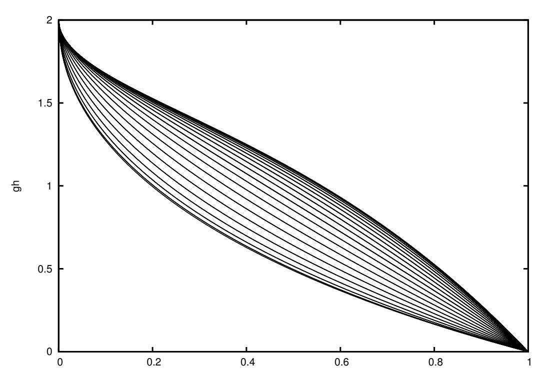


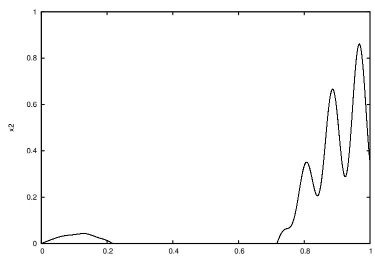
Fig. 4. POF of HE7 with $n_t = 10$ and $\tau_t = 10$ for 1000 iterations [16]. POF changes in a cyclic manner, moving from the middle to the top, then from the top to the middle, then from the middle to the bottom and then from the bottom to the middle. This whole process is then repeated.

Properties of HE7

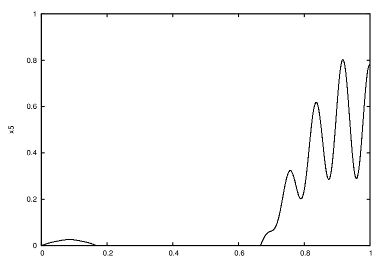
- Type III DMOOP
- POF changes from convex to concave, and vice versa
- POS (refer to Figure 5) and POF (refer to Figure 4) are:

$$POS: x_j = \begin{cases} a\cos\left(\frac{6\pi x_1 + \frac{j\pi}{n}}{3}\right), & j \in J_1 \\ a\sin\left(6\pi x_1 + \frac{j\pi}{n}\right), & j \in J_2 \\ \text{with:} & \\ a = \left[0.3x_1^2\cos(24\pi x_1 + \frac{4j\pi}{n}) + 0.6x_1\right] & \end{cases}$$

$$POF: y = (2-\sqrt{x_1})\left[1 - \left(\frac{x_1}{2-\sqrt{x_1}}\right)^{H(t)}\right]$$



(a) POS of x_2 with $n_t = 10$ and $\tau_t = 10$ for 1000 iterations



(b) POS of x_5 with $n_t = 10$ and $\tau_t = 10$ for 1000 iterations

Fig. 5. POS of HE7 for two decision variables, x_2 and x_5 [16]

HE9

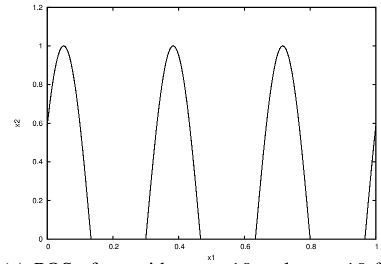
$$\text{HE9} = \begin{cases} Minimize: \mathbf{f}(\mathbf{x},t) = (f_1(\mathbf{x}), g(\mathbf{x},t)\cdot h(f_1(\mathbf{x}), \\ \qquad g(\mathbf{x},t))) \\ f_1(\mathbf{x}) = x_1 + \frac{2}{|J_1|}\sum_{j\in J_1}\left(x_j - \sin\left(6\pi x_1 + \frac{j\pi}{n}\right)\right)^2 \\ g(\mathbf{x}) = 2 - x_1^2 + \frac{2}{|J_2|}\sum_{j\in J_2}\left(x_j - \sin\left(6\pi x_1 + \frac{j\pi}{n}\right)\right)^2 \\ h(f_1,g) = 1 - \left(\frac{f_1}{g}\right)^{H(t)} \\ where: \\ H(t) = 0.75\sin(0.5\pi t) + 1.25, \quad t = \frac{1}{n_t}\left\lfloor\frac{\tau}{\tau_t}\right\rfloor \\ J_1 = \{j|\ \text{j is odd and } 2 \le j \le n\} \\ J_2 = \{j|\ \text{j is even and } 2 \le j \le n\} \\ x_i \in [0,1] \ \ \forall i = 1,2,\ldots,n \end{cases} \quad (12)$$

Properties of HE9

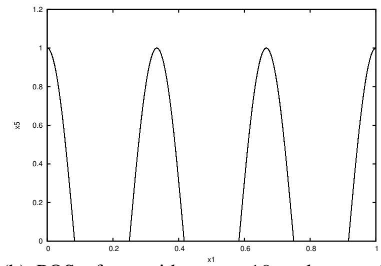
- Type III DMOOP
- POF changes from convex to concave, and vice versa
- POS (refer to Figure 6) and POF (refer to Figure 4) are:

$$POS: x_j = sin\left(6\pi x_1 + \frac{j\pi}{n}\right), \ \ \forall j = 2,3,\ldots,n.$$

$$POF: y = (2-\sqrt{x_1})\left[1 - \left(\frac{x_1}{2-\sqrt{x_1}}\right)^{H(t)}\right]$$



(a) POS of x_2 with $n_t = 10$ and $\tau_t = 10$ for 1000 iterations



(b) POS of x_5 with $n_t = 10$ and $\tau_t = 10$ for 1000 iterations

Fig. 6. POS of HE9 for two decision variables, x_2 and x_5

III. Experimental Setup

This section discusses the experimental setup that should be used for the competition.

A. Algorithm Settings

The values of parameters that should be used for the algorithm are presented in Table I.

TABLE I
$\#solutions$ AND $\#entities$ VALUES FOR THE ALGORITHM

$\#solutions$	$\#entities$
200	100

These values should be implemented in the algorithm as follows: At any iteration throughout the run an algorithm is not allowed to have more than 100 entities (individuals or particles). The maximum number of solutions in the approximated POF or archive is 200.

B. Benchmark Function Settings

This section presents the values of parameters that should be used for the benchmark functions.

1) Values for n_t and τ_t: Table II presents the 6 combinations of n_t and τ_t, and number of iterations (τ_T) that should be used for each of the benchmark functions listed in Section II-A. In Table II, n_t, τ_t and τ_T are the severity of change, frequency of change and maximum number of iterations respectively.

Therefore, for each n_t-τ_t combinations, there will be 20 environment changes.

2) Values for n and M: The values for n and M that should be used for the benchmark functions are indicated in Table III. In Table III, n and M are the number of decision variables and number of objective functions respectively.

Table IV presents the values that should be used for A, B and C for FDA5$_{iso}$, FDA5$_{dec}$, dMOP2$_{iso}$ and dMOP2$_{dec}$. In Table IV, A, B and C refer to the values of the mapping function, i.e. every decision variable between the values of B and C is mapped to the value A.

IV. Evaluation of DMOO Algorithms

This section discusses the process that will be followed to evaluate the performance of the algorithms. Section IV-A discusses the performance measure that will be used. The ranking of the algorithms is discussed in Section IV-B.

A. Performance Measure

Many of the existing performance measures' accuracy are affected when algorithms lose track of the changing POF or when outliers occur in the found POF. Based on the analysis done on existing performance measures by Helbig and Engelbrecht [17], [18], the following performance measure will be used to evaluate algorithms for this competition [23]:

$$acc_{alt}(t) = |HV(POF'(t)) - HV(POF^*(t))| \tag{13}$$

where HV is the hypervolume [24], [25].

TABLE II
n_t, τ_t and τ_T values for the benchmark functions

n_t	τ_t	τ_T
10	5	100
10	10	200
10	25	500
10	50	1000
1	10	200
1	50	1000
20	10	200
20	50	1000

TABLE III
n and M values for the benchmark functions

DMOOP	n	M
FDA4	12	3
FDA5	12	3
FDA5$_{iso}$	12	3
FDA5$_{dec}$	12	3
DIMP2	10	
dMOP2	10	
dMOP2$_{iso}$	10	
dMOP2$_{dec}$	10	
dMOP3	10	
HE2	30	
HE7	10	
HE10	10	

TABLE IV
A, B and C values for the benchmark functions FDA5$_{iso}$, FDA5$_{dec}$, dMOP2$_{iso}$ and dMOP2$_{dec}$

A	B	C
$G(t)$	0.001	0.05

B. Ranking of the Algorithms

The reference vector for the calculation of the HV will be calculated as the worst objective function values obtained from the set of all submitted data to the competition. For each time step just before a change in the environment occurs, the acc_{alt} value is calculated. After 30 runs, for each time step just before a change occurs, the average acc_{alt} value for the 30 runs is calculated. The calculation of wins and losses that is performed based on these acc_{alt} averaged values is presented in Algorithm 1 [26]. In Algorithm 1, $Diff = \#wins - \#losses$, where $Diff$ is the difference between the number of wins and number of losses assigned to the dynamic multi-objective optimisation algorithm (DMOA) and pm refers to performance measures values.These average values are then used to calculate the wins-and-losses for each algorithm as follows:

Algorithm 1 Calculation of wins and losses

```
for each DMOOP do
    for each n_t-τ_t combination do
        perform Kruskal-Wallis tests on pm
        if statistical significant difference then
            for each DMOA-pair do
                perform Mann-Whitney U test on pm
                if statistical significant difference then
                    assign wins and losses
                end if
            end for
        end if
    end for
    calculate Diff for the n_t-τ_t combination
end for
calculate Diff for the DMOOP
```

This approach takes into account the tracking ability of a DMOA. If the Mann-Whitney U test indicates that there is a significant difference, the average performance measure value of each time step just before a change in the environment occurred are used to award wins and losses. At each time step just before a change in the environment occurred, the average performance measure values of the two DMOAs are compared. The DMOA with the best performance measure value is awarded a win and the other DMOA is awarded a loss. In order to ensure that a DMOA that tracks the changing POF very well for a DMOOP does not lead to skewed results, the number of wins and losses are normalised as follows:

$$\begin{aligned} \#wins_{norm} &= \frac{\#wins}{\#changes} \\ \#losses_{norm} &= \frac{\#losses}{\#changes} \end{aligned} \tag{14}$$

where $\#changes$ represents the number of changes that occurred during the entire run. In situations where the time steps at which a change in the environment occurs are unknown, the algorithm should log detected changes during the run.

The total number of wins and losses for each algorithm is calculated and the algorithms are then ranked based on their $Diff$ value.

V. PROCESS TO ENTER COMPETITION

Participants should follow the following process to enter the competition:

- Please let us know if you intend to take part in the competition so that we can add you to our mailing list.
- Run your algorithm on the benchmark functions outlined in Section II, setting the parameter values of the functions and algorithm according to Section III.
- For each function, and for each iteration, record the found POF and POS.
- Send your POF and POS data files via email to: mgreeff@gmail.com or upload the files via Google docs and share the documents with the organizers. If you do not receive a confirmation email about your entry, contact one of the competition organizers.

It should be noted that by entering the competition you agree that your data can be shared with fellow researchers in the field. Therefore, this competition will provide DMOO researchers with experimental data to compare their algorithms against in future.

Furthermore, participants of the competition may also submit a paper to the associated special session, using their data submitted for this competition. However, the paper should include all information required to be reviewed independently for the special session.

The competition website is available at: https://sites.google.com/site/cec2015dmoocomp.

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REFERENCES

[1] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: NSGA-II," *IEEE Transactions on Evolutionary Computation*, vol. 6, no. 2, pp. 182–197, apr 2002.

[2] A. Konak, D. W. Coit, and A. E. Smith, "Multi-objective optimization using genetic algorithms: A tutorial," *Reliability Engineering & System Safety*, vol. 91, no. 9, pp. 992–1007, 2006.

[3] H. Tamaki, H. Kita, and S. Kobayashi, "Multi-objective optimization by genetic algorithms: a review," in *Proc. of IEEE International Conference on Evolutionary Computation*, 1996, pp. 517–522.

[4] C. C. Coello and M. Lechuga, "MOPSO: a proposal for multiple objective particle swarm optimization," in *Proceedings of Congress on Evolutionary Computation*, vol. 2, 2002, pp. 1051–1056.

[5] F. Campelo, F. G. Guimarães, and H. Igarashi, "Overview of artificial immune systems for multi-objective optimization," in *Evolutionary Multi-Criterion Optimization*, ser. Lecture Notes in Computer Science, S. Obayashi, K. Deb, C. Poloni, T. Hiroyasu, and T. Murata, Eds. Springer BerlinHeidelberg, 2007, vol. 4403, pp. 937–951.

[6] E. Mezura-Montes, M. Reyes-Sierra, and C. Coello, "Multi-objective optimization using differential evolution: A survey of the state-of-the-art," in *Advances in Differential Evolution*, ser. Studies in Computational Intelligence, U. Chakraborty, Ed. Springer BerlinHeidelberg, 2008, vol. 143, pp. 173–196.

[7] J. Branke, "Memory enhanced evolutionary algorithms for changing optimization problems," in *Proceedings of the Congress on Evolutionary Computation*, vol. 3, Washington DC, U.S.A., 1999, pp. 1875–1882.

[8] S. Yang, "Explicit memory schemes for evolutionary algorithms in dynamic environments," in *Evolutionary Computation in Dynamic and Uncertain Environments*, ser. Studies in Computational Intelligence, S. Yang, Y.-S. Ong, and Y. Jin, Eds. Springer Berlin/Heidelberg, 2007, vol. 51, pp. 3–28.

[9] ——, "Memory-based immigrants for genetic algorithms in dynamic environments," in *Proceedings of the 2005 conference on Genetic and Evolutionary Computation*, ser. GECCO '05, 2005, pp. 1115–1122.

[10] W. Du and B. Li, "Multi-strategy ensemble particle swarm optimization for dynamic optimization," *Information Sciences*, vol. 178, no. 15, pp. 3096–3109, 2008.

[11] T. Blackwell and J. Branke, "Multi-swarm optimization in dynamic environments," in *Applications of Evolutionary Computing*, ser. Lecture Notes in Computer Science, G. Raidl, S. Cagnoni, J. Branke, D. Corne, R. Drechsler, Y. Jin, C. Johnson, P. Machado, E. mariori, F. Rothlauf, G. Smith, and G. Squillero, Eds. Springer BerlinHeidelberg, 2004, vol. 3005, pp. 489–500.

[12] F. de Franca and F. Von Zuben, "A dynamic artificial immune algorithm applied to challenging benchmarking problems," in *Evolutionary Computation, 2009. CEC '09. IEEE Congress on*, 2009, pp. 423–430.

[13] J. Brest, A. Zamuda, B. Boskovic, M. Maucec, and V. Zumer, "Dynamic optimization using self-adaptive differential evolution," in *Proc. of IEEE Congress on Evolutionary Computation*, 2009, pp. 415–422.

[14] M. Helbig and A. Engelbrecht, "Challenges of dynamic multi-objective optimization," in *Proceedings of the BRICS Countries Congress on Computational Intelligence*, Porto de Galinhas, Brazil, sep 2013.

[15] ——, "Benchmarks for dynamic multi-objective optimisation algorithms," *ACM Computing Surveys*, 2013, in Press.

[16] ——, "Benchmarks for dynamic multi-objective optimisation," in *Proceedings of IEEE Symposium Series on Computational Intelligence*, Singapore, apr 2013, pp. 84–91.

[17] ——, "Performance measures for dynamic multi-objective optimisation algorithms," *Information Sciences*, vol. 250, pp. 61–81, 2013.

[18] ——, "Issues with performance measures for dynamic multi-objective optimisation," in *Proceedings of IEEE Symposium Series on Computational Intelligence*, Singapore, apr 2013, pp. 17–24.

[19] M. Farina, K. Deb, and P. Amato, "Dynamic multiobjective optimization problems: test cases, approximations, and applications," *IEEE Transactions on Evolutionary Computation*, vol. 8, no. 5, pp. 425–442, October 2004. [Online]. Available: http://www.cs.colostate.edu/ genitor/GECCO-2000/gecco2000mainpage.htm

[20] W. Koo, C. Goh, and K. Tan, "A predictive gradient strategy for multiobjective evolutionary algorithms in a fast changing environment," *Memetic Computing*, vol. 2, no. 2, pp. 87–110, 2010.

[21] C.-K. Goh and K. Tan, "A competitive-cooperative coevolutionary paradigm for dynamic multiobjective optimization," *IEEE Transactions on Evolutionary Computation*, vol. 13, no. 1, pp. 103–127, feb 2009.

[22] M. Helbig and A. Engelbrecht, "Archive management for dynamic multi-objective optimisation problems using vector evaluated particle swarm optimisation," in *Proceedings of Congress on Evolutionary Computation*, New Orleans, U.S.A., jun 2011, pp. 2047–2054.

[23] M. Cámara, J. Ortega, and F. de Toro, "Parallel processing for multi-objective optimization in dynamic environments," *International Parallel and Distributed Processing Symposium*, vol. 0, pp. 243–250, 2007.

[24] E. Zitzler and L. Thiele, "Multiobjective optimization using evolutionary algorithms a comparative case study," vol. 1498, pp. 292–301, 1998.

[25] ——, "Multiobjective evolutionary algorithms: a comparative case study and the strength pareto approach," *IEEE Transactions on Evolutionary Computation*, vol. 3, no. 4, pp. 257–271, nov 1999.

[26] M. Helbig and A. Engelbrecht, "Analysing the performance of dynamic multi-objective optimisation algorithms," in *Proceedings of IEEE Congress on Evolutionary Computation*, Canc/'un, Mexico, jun 2013, pp. 1531–1539.