Results of the 2015 IEEE CEC Competition on Niching Methods for Multimodal Optimization

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Outline

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Introduction

- Numerical optimization is probably one of the most important disciplines in optimization
- Many real-world problems are "multimodal" by nature, i.e., multiple satisfactory solutions exist
- Niching methods: promote and maintain formation of multiple stable subpopulations within a single population
  - Aim: maintain diversity and locate multiple globally optimal solutions.
- Challenge: Find an efficient optimization algorithm, which is able to locate multiple global optimal solutions for multimodal problems with various characteristics.

Competition

Provide a common platform that encourages fair and easy comparisons across different niching algorithms.

X. Li, A. Engelbrecht, and M.G. Epitropakis, "Benchmark Functions for CEC'2013 Special Session and Competition on Niching Methods for Multimodal Function Optimization", Technical Report, Evolutionary Computation and Machine Learning Group, RMIT University, Australia, 2013

- 20 benchmark multimodal functions with different characteristics
- 5 accuracy levels: $\varepsilon \in \{10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}\}$
- The benchmark suite and the performance measures have been implemented in: C/C++, Java, MATLAB, (Python soon)

Benchmark function set

X. Li, A. Engelbrecht, and M.G. Epitropakis, "Benchmark Functions for CEC'2013 Special Session and Competition on Niching Methods for Multimodal Function Optimization", Technical Report, Evolutionary Computation and Machine Learning Group, RMIT University, Australia, 2013

Id	Dim.	# GO	Name	Characteristics
F_1	1	2	Five-Uneven-Peak Trap	Simple, deceptive
F_2	1	5	Equal Maxima	Simple
F_3	1	1	Uneven Decreasing Maxima	Simple
F_4	2	4	Himmelblau	Simple, non-scalable, non-symmetric
F_5	2	2	Six-Hump Camel Back	Simple, not-scalable, non-symmetric
F_6	2,3	18,81	Shubert	Scalable, #optima increase with D, unevenly distributed grouped optima
F_7	2,3	36,216	Vincent	Scalable, #optima increase with D, unevenly distributed optima
F_8	2	12	Modified Rastrigin	Scalable, #optima independent from D, symmetric
F_9	2	6	Composition Function 1	Scalable, separable, non-symmetric
F_{10}	2	8	Composition Function 2	Scalable, separable, non-symmetric
F_{11}	2,3,5,10	6	Composition Function 3	Scalable, non-separable, non-symmetric
F_{12}	2,3,5,10	8	Composition Function 4	Scalable, non-separable, non-symmetric

Measures:

Peak Ratio (PR) measures the average percentage of all known global optima found over multiple runs:

$$\text{PR} = \frac{\sum_{run=1}^{NR} \#\text{ of Global Optima}_i}{(\#\text{ of known Global Optima}) * (\#\text{ of runs})}$$

Who is the winner:

- The participant with the highest average Peak Ratio performance on all benchmarks wins.
- In all functions the following holds: the higher the PR value, the better

Participants

Submissions to the competition:

- (**ALNM**) An Active Learning Based Niching Method with Sequential Binary Probabilistic Classification and Class Split Threshold Updating, (Yuqing Zhou and Kazuhiro Saitou) [1]
- (**MEA**) Multinational Evolutionary Algorithm of Ursem [2]
- (**MSSPSO**) Multi-Sub-Swarm Particle Swarm Optimisation Algorithm of Zhang et al. [3]
- (**LSEAGP**) Localised Search Evolutionary Algorithm using a Gaussian Process of Fieldsend [4];
- (**LSEAEA**) Localised Search Evolutionary Algorithm using an Evolutionary Algorithm of Fieldsend [5];
- (**NMMSO**) Niching Migratory Multi-Swarm Optimiser of Fieldsend [6].

Participants (2)

Implemented algorithms for comparisons:

- (**CrowdingDE**) Crowding Differential Evolution [7]
- (**DE/nrand/1**) Niching Differential Evolution algorithms with neighborhood mutation strategies [8]
- (**dADE/nrand/1**) A Dynamic Archive Niching Differential Evolution algorithm for Multimodal Optimization [9]
- (**NEA2**) Niching the CMA-ES via Nearest-Better Clustering [10]

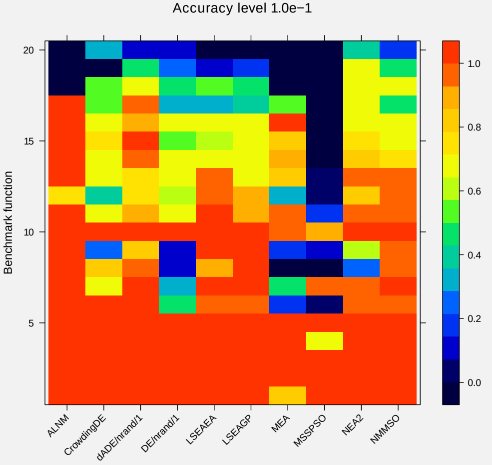
In the repository: CMA-ES, IPOP-CMA-ES, DE/nrand/1,2, DECG, DELG, DELS-aj, CrowdingDE, dADE/nrand/1,2, NEA1, NEA2, N-VMO, PNA-NSGAII, A-NSGAII

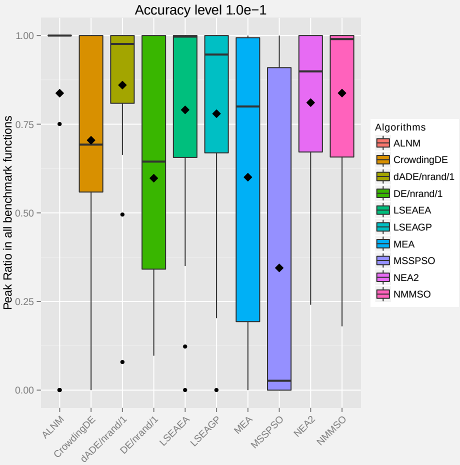
Results

Summary:

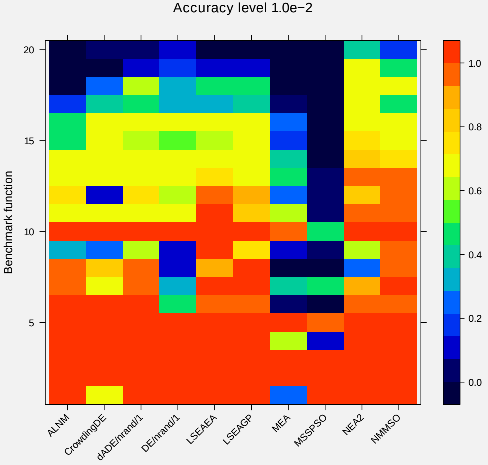
- **6** new search algorithms
- **4** comparators based on the competition @ CEC2013
- **20** multi-modal benchmark functions
- **5** accuracy levels $\varepsilon \in \{10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}\}$
- Results: **per accuracy level & over all accuracy levels**
- In total (CEC2013 & CEC2015) 21 algorithms in the repository: `https://github.com/mikeagn/CEC2013`

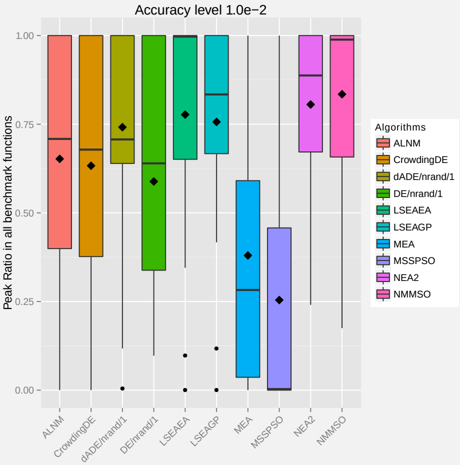
Accuracy level $\varepsilon = 10^{-1}$



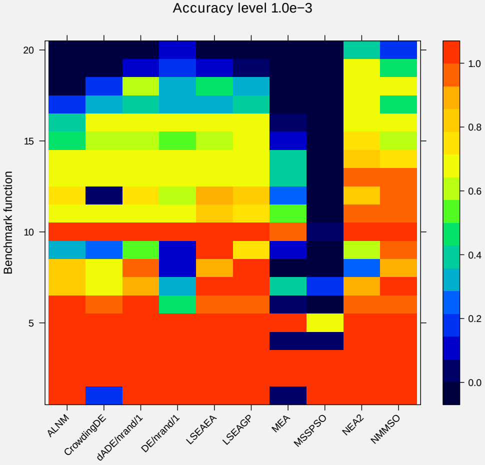


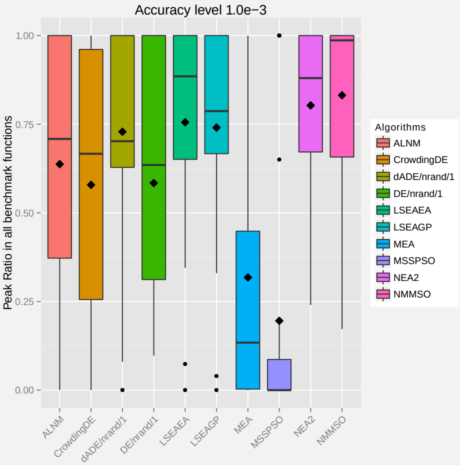
Accuracy level $\varepsilon = 10^{-2}$



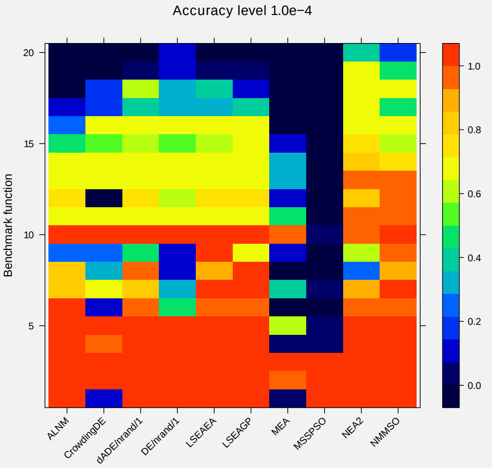


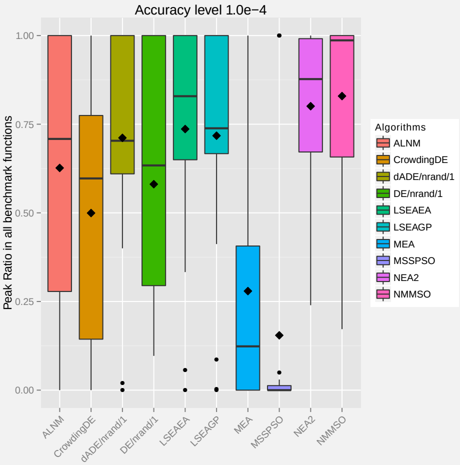
Accuracy level $\varepsilon = 10^{-3}$



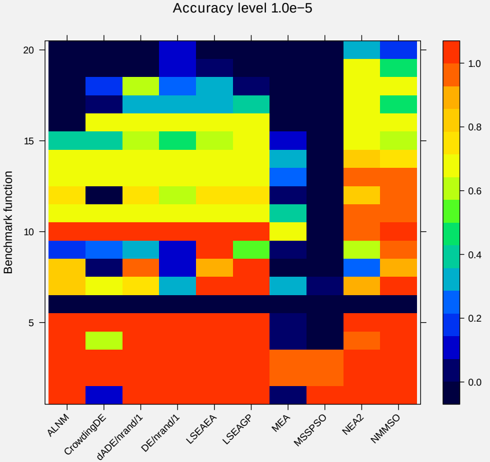


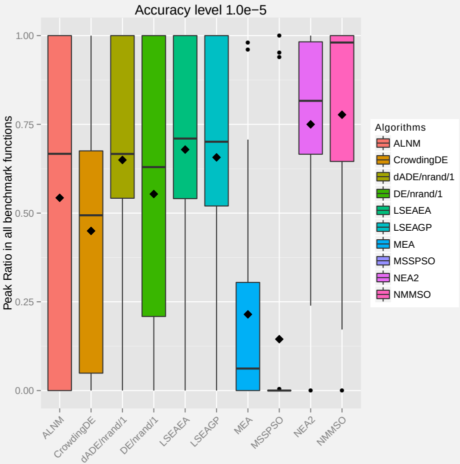
Accuracy level $\varepsilon = 10^{-4}$



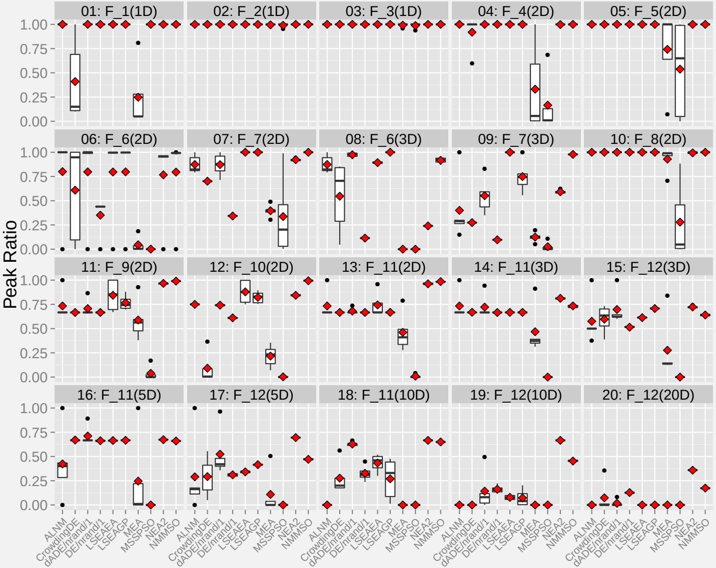


Accuracy level $\varepsilon = 10^{-5}$

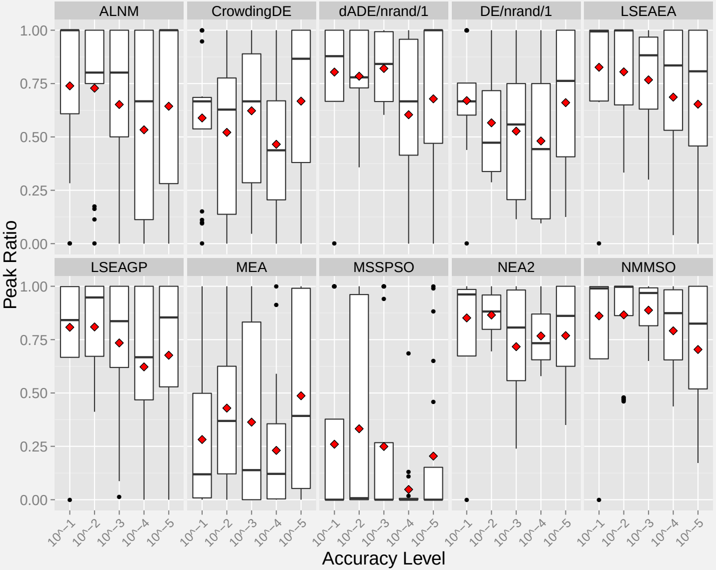




Performance per benchmark across all accuracy levels



Performance per algorithm

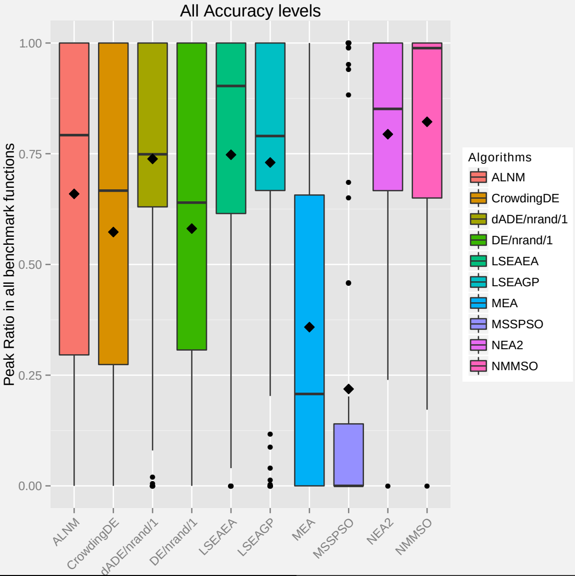


Statistical Analysis

	ALNM	CrowdingDE	dADE/nrand/1	DE/nrand/1	LSEAEA	LSEAGP	MEA	MSSPSO	NEA2
	p/p_b	p/p_b	p/p_b	p/p_b	p/p_b	p/p_b	p/p_b	p/p_b	p/p_b
CrowdingDE	–/=	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
dADE/nrand/1	–/–	**+/+**	N/A	N/A	N/A	N/A	N/A	N/A	N/A
DE/nrand/1	–/=	=/=	–/–	N/A	N/A	N/A	N/A	N/A	N/A
LSEAEA	**+/+**	**+/+**	=/=	**+/+**	N/A	N/A	N/A	N/A	N/A
LSEAGP	–/–	**+/+**	=/=	**+/+**	=/=	N/A	N/A	N/A	N/A
MEA	–/–	–/–	–/–	–/–	–/–	–/–	N/A	N/A	N/A
MSSPSO	–/–	–/–	–/–	–/–	–/–	–/–	–/–	N/A	N/A
NEA2	**+/+**	**+/+**	**+/+**	**+/+**	–/=	**+**/=	**+/+**	**+/+**	N/A
NMMSO	**+/+**	**+/+**	**+/+**	**+/+**	**+/+**	**+/+**	**+/+**	**+/+**	=/=

- p: Wilcoxon rank-sum test
- p_b: Bonferroni correction
- **+** row wins column,
- **–** row loses from column,
- **=** non-significant differences
- N/A: Not Applicable

Overall performance (1)



Overall performance (2)

	Algorithm	Statistics			Friedman's Test	
		Median	Mean	St.D.	Rank	Score
CEC2013	CrowdingDE	0.6667	0.5731	0.3612	8	4.8150
	DE/nrand/1	0.6396	0.5809	0.3338	7	4.8250
	dADE/nrand/1	0.7488	0.7383	0.3010	5	6.3250
	NEA2	**0.8513**	**0.7940**	0.2332	**2**	**7.4800**
CEC2015	ALNM	0.7920	0.6594	0.3897	6	5.6700
	LSEAEA	**0.9030**	**0.7477**	0.3236	**4**	**6.6000**
	LSEAGP	**0.7900**	**0.7302**	0.3268	**3**	**6.7500**
	MEA	0.2075	0.3585	0.3852	9	2.8550
	MSSPSO	0.0000	0.2188	0.3913	10	2.1400
	NMMSO	**0.9885**	**0.8221**	0.2538	**1**	**7.5400**

Overall performance (3) CEC2013 + CEC2015

Algorithm	Statistics			Friedman's Test	
	Median	Mean	St.D.	Rank	Score
NMMSO	**0.9885**	**0.8221**	0.2538	**1**	16.1900
NEA2	**0.8513**	**0.7940**	0.2332	**2**	16.1150
LSEAEA	**0.9030**	**0.7477**	0.3236	**4**	14.5050
dADE/nrand/1	0.7488	**0.7383**	0.3010	5	14.2450
LSEAGP	**0.7900**	0.7302	0.3268	**3**	14.7550
CMA-ES	0.7550	0.7137	0.2807	6	14.0800
N-VMO	0.7140	0.6983	0.3307	7	13.7600
ALNM	0.7920	0.6594	0.3897	9	12.4900
PNA-NSGAII	0.6660	0.6141	0.3421	11	11.2700
NEA1	0.6496	0.6117	0.3280	14	10.5250
DE/nrand/2	0.6667	0.6082	0.3130	10	11.2950
dADE/nrand/2	0.7150	0.6931	0.3174	8	12.8100
DE/nrand/1	0.6396	0.5809	0.3338	13	10.6150
DELS-aj	0.6667	0.5760	0.3857	15	9.6950
CrowdingDE	0.6667	0.5731	0.3612	12	10.6200
DELG	0.6667	0.5706	0.3925	16	9.4400
DECG	0.6567	0.5516	0.3992	17	8.9900
IPOP-CMA-ES	0.2600	0.3625	0.3117	18	5.8700
MEA	0.2075	0.3585	0.3852	19	5.2750
A-NSGAII	0.0740	0.3275	0.4044	20	4.7200
MSSPSO	0.0000	0.2188	0.3913	21	3.7350

Winners

Ranking based on average PR values (only CEC2015)

1. **NMMSO:** Niching Migratory Multi-Swarm Optimiser [6].
2. **LSEAGP:** Localised Search Evolutionary Algorithm using a Gaussian Process [4]
3. **LSEAEA:** Localised Search Evolutionary Algorithm using an Evolutionary Algorithm [5]
4. **ALNM:** An Active Learning Based Niching Method with Sequential Binary Probabilistic Classification and Class Split Threshold Updating [1]

Note: The algorithms have not been fine-tuned for the specific benchmark suite!

Note: NEA2 rank 2, dADE/nrand/1 rank 4 based on average PR values

Conclusions

Summary

- Six new search algorithms (in total 21 algorithms!)
- Winner: **NMMSO:** Niching Migratory Multi-Swarm Optimiser [6].
  - Competitive on average performance, (multi-swarms, migration, sub-swarm merging, PSO)
- Places 2 to 4:
  - **LSEAGP:** Localised Search Evolutionary Algorithm using a Gaussian Process [4]
  - **LSEAEA:** Localised Search Evolutionary Algorithm using an Evolutionary Algorithm [5]
  - **ALNM:** An Active Learning Based Niching Method with Sequential Binary Probabilistic Classification and Class Split Threshold Updating [1]

Conclusions (2)

- The competition gave a boost to the multimodal optimization community
- New competitive and very promising approaches

Key characteristics of the algorithms:

- New methodologies, (active learning, surrogates, Gaussian Processes, probabilistic classifier for prediction, hill-valley approaches)
- Usage of local models to maintain diversity and exploit locally the neighborhoods
- Algorithms: Evolutionary Algorithms, Multi-swarms, and Bootstrap-LV sampling.

Future Work

Possible objectives:

- Re-organize the competitions in future
- Enhance the benchmark function set
- Introduce new performance measures
- Automate the experimental design and results output
- Boost multimodal optimization community

Acknowledgment

We really want to thank for their help:

- The participants :-)
- Prof. Fabrício Olivetti de França & Eduardo Nobre (python initial implementation)

(-: Thank you very much for your attention :-)

Questions ???

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