Problem Definitions and Evaluation Criteria for the CEC 2015 Competition on Single Objective Multi-Niche Optimization

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Please Note that in this competition error values smaller than 10^-8 will be taken as zero.

You can download the C, JAVA and Matlab codes for CEC'15 niching test suite from the website given below:

http://www.ntu.edu.sg/home/EPNSugan/index_files/CEC2015/CEC2015.htm

This technical report presents the details of benchmark suiteused for CEC'15bound constrained single objective multi-niche optimization problems. For single objective bound constrained expensive optimization problems and learning based optimization, please refer to [1][2].

1. Introduction to the CEC'15Multi-NicheBenchmark Suite

1.1 Some Definitions:

All test functions are minimization problems defined as follows:

$$\text{Min } f(\mathbf{x}),\ \boldsymbol{x} = [x_1, x_2, ..., x_D]^{\mathrm{T}}$$

D: dimensions.
q: the goal optima number.

$\boldsymbol{o}_{i1} = [o_{i1}, o_{i2}, ..., o_{iD}]^{\mathrm{T}}$: the shifted global optimum (defined in "shift_data_x.txt"), which is randomly distributed in $[-80,80]^D$. Each function has a shift data for CEC'15.

All test functions are shifted to $\boldsymbol{o}$ and scalable (Some test functions are only scalable for even dimensions).

For convenience, the same search ranges are defined for all test functions.

Search range: $[-100,100]^D$.

$\mathbf{M}_i$: rotation matrix. Different rotation matrixes are assigned to each function and each basic function.

The variables are divided into subcomponents randomly. The rotation matrix for each subcomponents are generated from standard normally distributed entries by Gram-Schmidt ortho-normalization with condition number *c* that is equal to 1.

1.2 Summary of the CEC'15 Multi-Niche Test Suite

Table I. Summary of the CEC'15 CEC'15 Multi-Niche Test Functions

<table>
<tr><th></th><th>No.</th><th>Functions</th><th>Dimension</th><th>Goal optima No. global/local*</th><th>F_i*=$F_i(x^*)$</th></tr>
<tr><td rowspan="24">Expanded Scalable Function</td><td rowspan="3">1</td><td rowspan="3">Shifted and Rotated Expanded Two-Peak Trap</td><td>5</td><td>1/15</td><td rowspan="3">100</td></tr>
<tr><td>10</td><td>1/55</td></tr>
<tr><td>20</td><td>1/210</td></tr>
<tr><td rowspan="3">2</td><td rowspan="3">Shifted and Rotated Expanded Five-Uneven-Peak Trap</td><td>2</td><td>4/21</td><td rowspan="3">200</td></tr>
<tr><td>5</td><td>32/0</td></tr>
<tr><td>8</td><td>256/0</td></tr>
<tr><td rowspan="3">3</td><td rowspan="3">Shifted and Rotated Expanded Equal Minima</td><td>2</td><td>25/0</td><td rowspan="3">300</td></tr>
<tr><td>3</td><td>125/0</td></tr>
<tr><td>4</td><td>625/0</td></tr>
<tr><td rowspan="3">4</td><td rowspan="3">Shifted and Rotated Expanded Decreasing Minima</td><td>5</td><td>1/15</td><td rowspan="3">400</td></tr>
<tr><td>10</td><td>1/55</td></tr>
<tr><td>20</td><td>1/210</td></tr>
<tr><td rowspan="3">5</td><td rowspan="3">Shifted and Rotated Expanded Uneven Minima</td><td>2</td><td>25/0</td><td rowspan="3">500</td></tr>
<tr><td>3</td><td>125/0</td></tr>
<tr><td>4</td><td>625/0</td></tr>
<tr><td rowspan="3">6</td><td rowspan="3">Shifted and Rotated Expanded Himmelblau's Function</td><td>4</td><td>16/0</td><td rowspan="3">600</td></tr>
<tr><td>6</td><td>64/0</td></tr>
<tr><td>8</td><td>256/0</td></tr>
<tr><td rowspan="3">7</td><td rowspan="3">Shifted and Rotated Expanded Six-Hump Camel Back</td><td>6</td><td>8/0</td><td rowspan="3">700</td></tr>
<tr><td>10</td><td>32/0</td></tr>
<tr><td>16</td><td>256/0</td></tr>
<tr><td rowspan="3">8</td><td rowspan="3">Shifted and Rotated Modified Vincent Function</td><td>2</td><td>36/0</td><td rowspan="3">800</td></tr>
<tr><td>3</td><td>216/0</td></tr>
<tr><td>4</td><td>1296/0</td></tr>
<tr><td rowspan="7">Composition Function</td><td>9</td><td>Composition Function 1</td><td>10, 20, 30</td><td>10/0</td><td>900</td></tr>
<tr><td>10</td><td>Composition Function 2</td><td>10, 20, 30</td><td>1/9</td><td>1000</td></tr>
<tr><td>11</td><td>Composition Function 3</td><td>10, 20, 30</td><td>10/0</td><td>1100</td></tr>
<tr><td>12</td><td>Composition Function 4</td><td>10, 20, 30</td><td>10/0</td><td>1200</td></tr>
<tr><td>13</td><td>Composition Function 5</td><td>10, 20, 30</td><td>10/0</td><td>1300</td></tr>
<tr><td>14</td><td>Composition Function 6</td><td>10, 20, 30</td><td>1/19</td><td>1400</td></tr>
<tr><td>15</td><td>Composition Function 7</td><td>10, 20, 30</td><td>1/19</td><td>1500</td></tr>
<tr><td colspan="6">Search Range: [-100,100]^D
level of accuracy = 0.1</td></tr>
</table>

***Please Note:**

1. These problems should be treated as **black-box problems**. The explicit equations of the problems are not to be used.

2. 2. Goal Peaks **global/local** here is the total number of global and local solutions required. Some test functions have numerous poor quality local optima while algorithms are expected to capture the best local solutions as required.

1.3 Definitions of the Basic Functions

1.3. 1 Novel Expanded Multi-Niche Problems:

1) Expanded Two-Peak Trap

$$f_1(\boldsymbol{x}) = \sum_{i=1}^{D} t_i + 200D$$

$$t_i = \begin{cases} -160 + y_i^2, & \text{if } y_i < 0 \\ \dfrac{160}{15}(y_i - 15), & \text{if } 0 \le y_i \le 15 \\ \dfrac{200}{5}(15 - y_i), & \text{if } 15 \le y_i \le 20 \\ -200 + (y_i - 20)^2, & \text{if } y_i > 20 \end{cases}$$

$$\boldsymbol{y} = \boldsymbol{x} + 20$$

$$f_1(\boldsymbol{x}^*) = 0, \quad \boldsymbol{x}^* = [0, 0, ..., 0]^D$$

Table II. No. of optima for Expanded Two-Peak Trap

Type of Optima	No.
Global Optimum	1
Local Optima	$2^D - 1$
Second and Third Best Optima	$C_D^1 + C_D^2$

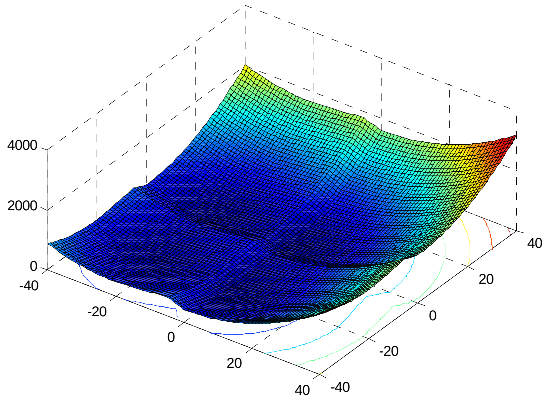

Figure 1(a).3-*D* map for 2-*D* function $f_1(\boldsymbol{x})$

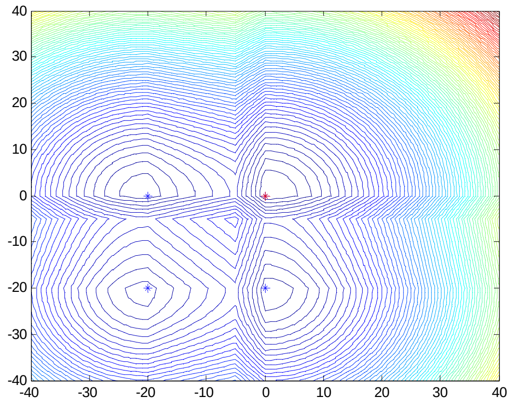

Figure 1(b). Contour map for 2-*D* function $f_1(\boldsymbol{x})$

2) Expanded Five-Uneven-Peak Trap

$$f_2(\boldsymbol{x}) = \sum_{i=1}^{D} t_i + 200D$$

$$t_i = \begin{cases} -200 + x_i^2 & \text{if } x_i < 0 \\ -80(2.5 - x_i) & \text{if } 0 \le x_i < 2.5 \\ -64(x_i - 2.5) & \text{if } 2.5 \le x_i < 5 \\ -64(7.5 - x_i) & \text{if } 5 \le x_i < 7.5 \\ -28(x_i - 7.5) & \text{if } 7.5 \le x_i < 12.5 \\ -28(17.5 - x_i) & \text{if } 12.5 \le x_i < 17.5 \\ -32(x_i - 17.5) & \text{if } 17.5 \le x_i < 22.5 \\ -32(27.5 - x_i) & \text{if } 22.5 \le x_i < 27.5 \\ -80(x_i - 27.5) & \text{if } 27.5 \le x_i \le 30 \\ -200 + (x_i - 30)^2 & \text{if } x_i > 30 \end{cases}$$

$$f_2(\boldsymbol{x}^*) = 0, \quad \boldsymbol{x}_i^* = 0 \text{ or } 30 \ \text{ for } i = 1, 2, \ldots, D$$

Table III. No. of optima for Expanded Five-Uneven-Peak Trap

Type of Optima	No.
Global Optima	2^D
Local Optima	$5^D - 2^D$

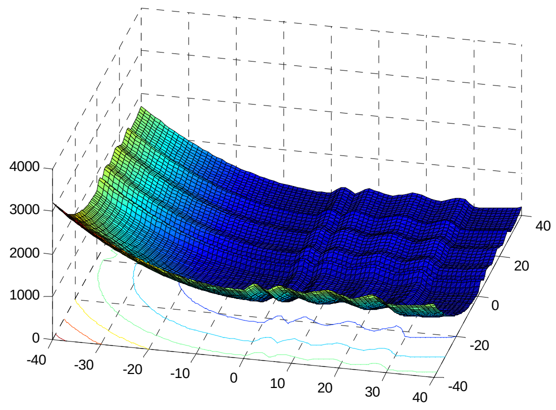

Figure 2(a).3-D map for 2-D function $f_2(\boldsymbol{x})$

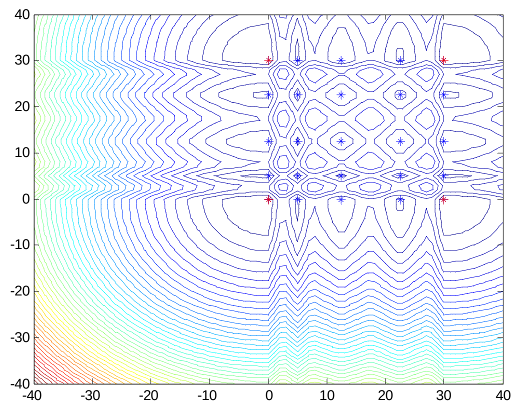

Figure 2(b).Contour map for 2-D function $f_2(\boldsymbol{x})$

3) Expanded Equal Minima

$$f_3(\boldsymbol{x}) = \sum_{i=1}^{D} t_i + D$$

$$t_i = \begin{cases} y_i^2 & \text{if } y_i < 0 \text{ or } y_i > 1 \\ -\sin^6(5\pi y_i) & \text{if } 0 \le y_i \le 1 \end{cases}, \qquad i = 1, 2, \ldots, D$$

$$\boldsymbol{y} = \boldsymbol{x} + 0.1$$

$$f_3(\boldsymbol{x}^*) = 0, \quad \boldsymbol{x}_i^* = 0.0,\ 0.2,\ 0.4,\ 0.6 \text{ or } 0.8 \ \text{ for } i = 1, 2, \ldots, D$$

Table IV. No. of optima for Expanded Equal Minima

Type of Optima	No.
Global Optima	5^D
Local Optima	0

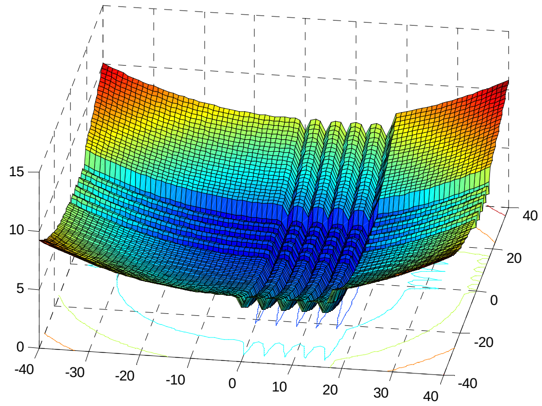

Figure 3(a).3-D map for 2-D function $f_3(\boldsymbol{x})$

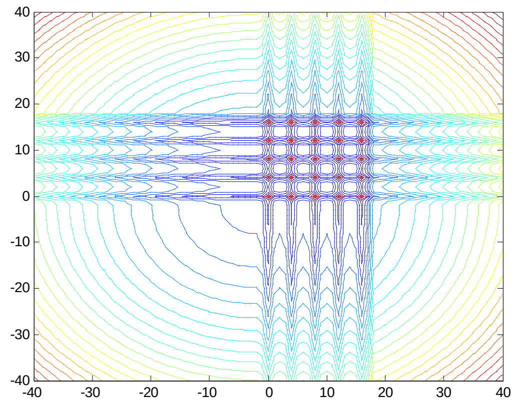

Figure 3(b).Contour map for 2-D function $f_3(\boldsymbol{x})$

4) Expanded Decreasing Minima

$$f_4(\boldsymbol{x}) = \sum_{i=1}^{D} t_i + D$$

$$t_i = \begin{cases} y_i^2 & \text{if } y_i < 0 \text{ or } y_i > 1 \\ -\exp[-2\log(2)\cdot(\frac{y_i - 0.1}{0.8})^2]\cdot\sin^6(5\pi y_i) & \text{if } 0 \le y_i \le 1 \end{cases},$$

$$i = 1, 2, ..., D$$

$$\boldsymbol{y} = \boldsymbol{x} + 0.1$$

$$f_4(\boldsymbol{x}^*) = 0, \quad \boldsymbol{x}^* = [0,0,...,0]^D$$

Table V. No. of optima for Expanded Decreasing Minima

Type of Optima	No.
Global Optimum	1
Local Optima	$5^D - 1$
Second and Third Best Optima	$C_D^1 + C_D^2$

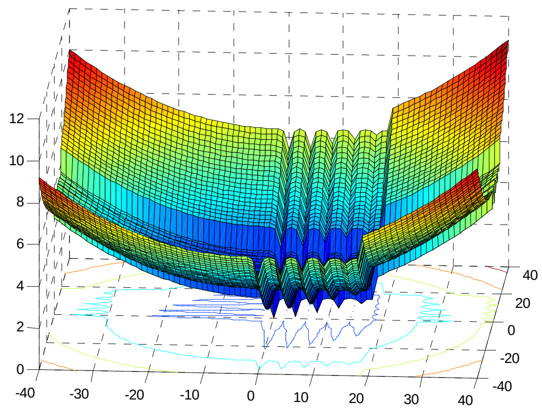

Figure 4(a).3-D map for 2-D function $f_4(\boldsymbol{x})$

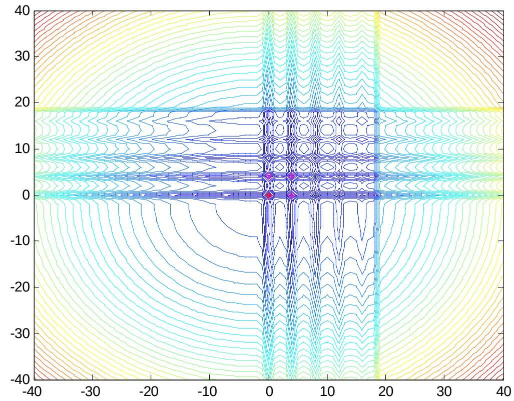

Figure 4(b).Contour map for 2-D function $f_4(\boldsymbol{x})$

5) Expanded Uneven Minima

$$f_5(\boldsymbol{x}) = \sum_{i=1}^{D} t_i - D$$

$$t_i = \begin{cases} y_i^2 & \text{if } y_i < 0 \text{ or } y_i > 1 \\ -\sin^6(5\pi(y_i^{3/4} - 0.05)) & \text{if } 0 \le y_i \le 1 \end{cases}, \quad i = 1,2,...,D$$

$$\boldsymbol{y} = \boldsymbol{x} + 0.079699392688696$$

$$f_5(\boldsymbol{x}^*) = 0, \quad \boldsymbol{x}_i^* = \begin{cases} 0 \\ \text{or } 0.166955 \\ \text{or } 0.370927 \\ \text{or } 0.601720 \\ \text{or } 0.854195 \end{cases} \quad \text{for } i = 1, 2, \ldots, D$$

Table VI. No. of optima for Expanded Uneven Minima

Type of Optima	No.
Global Optima	5^D
Local Optima	0

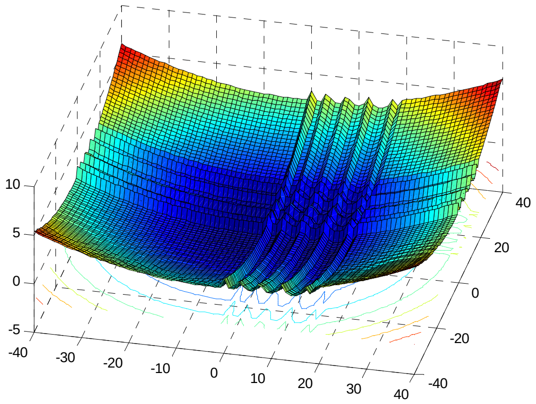

Figure 5(a).3-D map for 2-D function $f_5(\boldsymbol{x})$

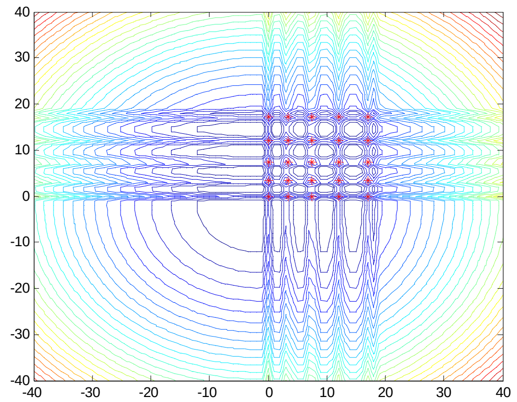

Figure 5(b).Contour map for 2-D function $f_5(\boldsymbol{x})$

6) Expanded Himmelblau's Function

$$f_5(x) = \sum_{i=1,3,5,\ldots}^{D-1} [(y_i^2 + y_{i+1} - 11)^2 + (y_i + y_{i+1}^2 - 7)^2]$$

$$y_i = \begin{cases} x_i - 3, & \text{if } i \text{ is odd number} \\ x_i - 2, & \text{if } i \text{ is even number} \end{cases}, \qquad i = 1, 2, ..., D$$

D must be an even number.

$$f_6(\boldsymbol{x^*}) = 0$$
$$\boldsymbol{x^*} = [\boldsymbol{y}_1, ..., \boldsymbol{y}_{D/2}]$$
$$\boldsymbol{y}_i = \begin{cases} [0, 0] \\ \text{or } [0.584428, \text{-}3.848126] \\ \text{or } [\text{-}6.779310, \text{-}5.283186] \\ \text{or } [\text{-}5.805118, 1.131312] \end{cases} \qquad \text{for } i = 1, 2, ..., \frac{D}{2}$$

Table VII. No. of optima for Expanded Himmelblau's Function

Type of Optima	No.
Global Optima	$4^{D/2}$
Local Optima	0

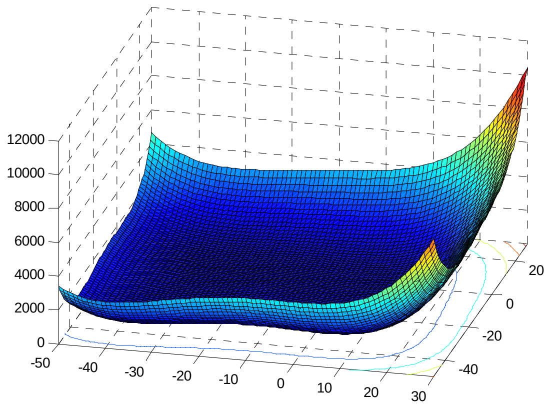

Figure 6(a).3-D map for 2-D function $f_6(\boldsymbol{x})$

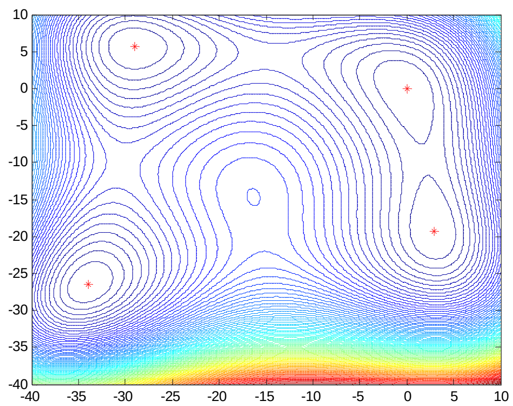

Figure 6(b).Contour map for 2-D function $f_6(\boldsymbol{x})$

7) Expanded Six-Hump Camel Back

$$f_7(\boldsymbol{x}) = \sum_{i=1,3,5,...}^{D-1} \{-4[(4-2.1y_i^2 + \frac{y_i^4}{3})y_i^2 + y_i y_{i+1} + (-4+4y_{i+1}^2)y_{i+1}^2]\}$$

$$y_i = \begin{cases} x_i - 0.089842, & \text{if } i \text{ is odd number} \\ x_i + 0.712656, & \text{if } i \text{ is even number} \end{cases}, \qquad i = 1, 2, ..., D$$

D must be an even number

$$f_7(\boldsymbol{x^*}) = 0$$

$$\boldsymbol{x^*} = [\boldsymbol{y}_1, ..., \boldsymbol{y}_{D/2}]$$

$$\boldsymbol{y}_i = \begin{cases} [0, 0] \\ \text{or } [-0.179684, 1.425312] \end{cases} \qquad \text{for } i = 1, 2, ..., \frac{D}{2}$$

Table VIII. No. of optima for Expanded Six-Hump Camel Back

Type of Optima	No.
Global Optima	$2^{D/2}$
Local Optima	0

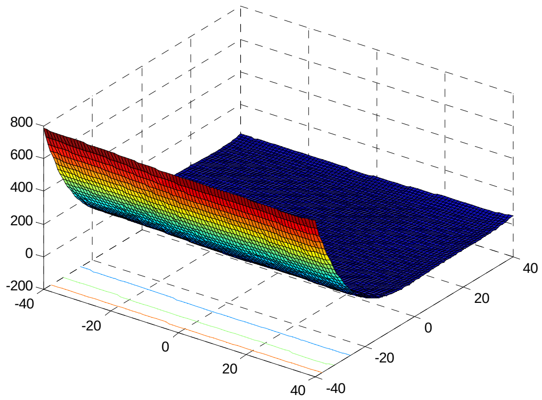

Figure 7(a).3-D map for 2-D function $f_7(\boldsymbol{x})$

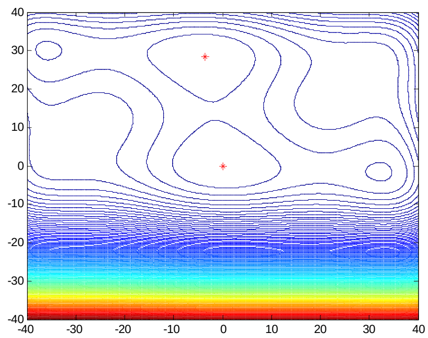

Figure 7(b).Contour map for 2-D function $f_7(\boldsymbol{x})$

8) Modified Vincent Function

$$f_8(\boldsymbol{x}) = \frac{1}{D}\sum_{i=1}^{D}(t_i + 0.1)$$

$$t_i = \begin{cases} \sin(10\log(y_i)) & \text{if } 0.25 \le y_i \le 10 \\ (0.25 - y_i)^2 + \sin(10\log(2.5)) & \text{if } y_i < 0.25 \\ (y_i - 10)^2 + \sin(10\log(10)) & \text{if } y_i > 10 \end{cases}, \quad i = 1, 2, \ldots, D$$

$$\boldsymbol{y} = \boldsymbol{x} + 4.1112$$

$$f_8(\boldsymbol{x}^*) = 0, \quad \boldsymbol{x}_i^* = \begin{cases} \quad -3.7782 \\ \text{or } -3.4870 \\ \text{or } -2.9411 \\ \text{or } -1.9179 \\ \text{or } 0 \\ \text{or } 3.5951 \end{cases} \quad \text{for } i = 1, 2, \ldots, D$$

Table IX. No. of optima for Modified Vincent Function

Type of Optima	No.
Global Optima	6^D
Local Optima	0

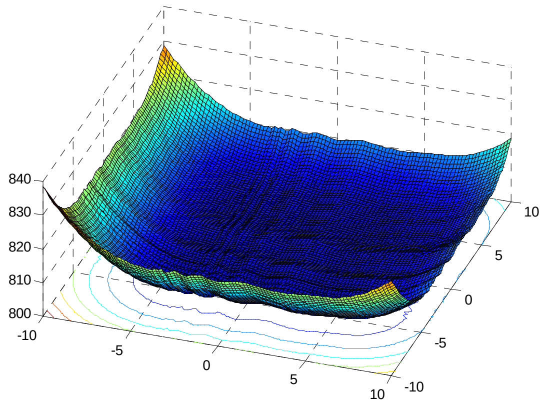

Figure 8(a).3-D map for 2-D function $f_8(\boldsymbol{x})$

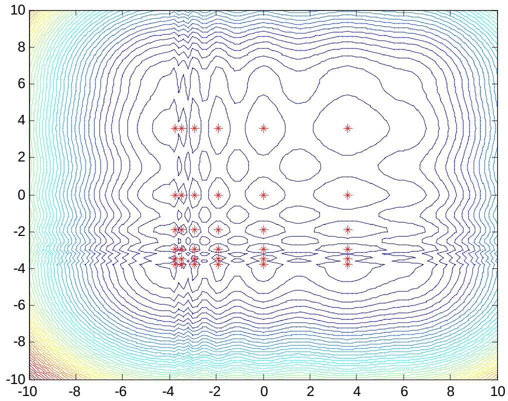

Figure 8(b). Contour map for 2-D function $f_8(\boldsymbol{x})$

1.3. 2 Basic Functions for Constructing Composition Functions

1) Sphere Function

$$f_9(\boldsymbol{x}) = \sum_{i=1}^{D} x_i^{\ 2}$$

2) High Conditioned Elliptic Function

$$f_{10}(\boldsymbol{x}) = \sum_{i=1}^{D} (10^6)^{\frac{i-1}{D-1}} \boldsymbol{x}_i^2$$

3) Bent Cigar Function

$$f_{11}(\boldsymbol{x}) = x_1^2 + 10^6 \sum_{i=2}^{D} x_i^2$$

4) Discus Function

$$f_{12}(\boldsymbol{x}) = 10^6 x_1^2 + \sum_{i=2}^{D} x_i^2$$

5) Different Powers Function

$$f_{13}(\boldsymbol{x}) = \sqrt{\sum_{i=1}^{D} \left| x_i \right|^{2+4\frac{i-1}{D-1}}}$$

6) Rosenbrock's Function

$$f_{14}(\boldsymbol{x}) = \sum_{i=1}^{D-1} (100(x_i^2 - x_{i+1})^2 + (x_i - 1)^2)$$

7) Ackley's Function

$$f_{15}(\boldsymbol{x}) = -20\exp(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D} x_i^2}) - \exp(\frac{1}{D}\sum_{i=1}^{D}\cos(2\pi x_i)) + 20 + e$$

8) Weierstrass Function

$$f_{16}(\boldsymbol{x}) = \sum_{i=1}^{D}(\sum_{k=0}^{k\max}[a^k \cos(2\pi b^k (x_i + 0.5))]) - D\sum_{k=0}^{k\max}[a^k \cos(2\pi b^k \cdot 0.5)]$$

a=0.5, b=3, $kmax$=20

9) Griewank's Function

$$f_{17}(\boldsymbol{x}) = \sum_{i=1}^{D}\frac{x_i^{\,2}}{4000} - \prod_{i=1}^{D}\cos(\frac{x_i}{\sqrt{i}}) + 1$$

10) Rastrigin's Function

$$f_{18}(\boldsymbol{x}) = \sum_{i=1}^{D}(x_i^{\,2} - 10\cos(2\pi x_i) + 10)$$

11) Modified Schwefel's Function

$$f_{19}(\boldsymbol{x}) = 418.9829 \times D - \sum_{i=1}^{D} g(z_i), \qquad z_i = x_i + 4.209687462275036\text{e}{+}002$$

$$g(z_i) = \begin{cases} z_i \sin(|z_i|^{1/2}) & \text{if } |z_i| \le 500 \\ (500 - \text{mod}(z_i, 500))\sin(\sqrt{|500 - \text{mod}(z_i, 500)|}) - \dfrac{(z_i - 500)^2}{10000D} & \text{if } z_i > 500 \\ (\text{mod}(|z_i|, 500) - 500)\sin(\sqrt{|\text{mod}(|z_i|, 500) - 500|}) - \dfrac{(z_i + 500)^2}{10000D} & \text{if } z_i < -500 \end{cases}$$

12) Katsuura Function

$$f_{20}(\boldsymbol{x}) = \frac{10}{D^2}\prod_{i=1}^{D}(1 + i\sum_{j=1}^{32}\frac{|2^j x_i - round(2^j x_i)|}{2^j})^{\frac{10}{D^{1.2}}} - \frac{10}{D^2}$$

13) HappyCat Function

$$f_{21}(\mathbf{x}) = \left| \sum_{i=1}^{D} x_i^2 - D \right|^{1/4} + (0.5\sum_{i=1}^{D} x_i^2 + \sum_{i=1}^{D} x_i)/D + 0.5$$

14) HGBat Function

$$f_{22}(\mathbf{x}) = \left| (\sum_{i=1}^{D} x_i^2)^2 - (\sum_{i=1}^{D} x_i)^2 \right|^{1/2} + (0.5\sum_{i=1}^{D} x_i^2 + \sum_{i=1}^{D} x_i)/D + 0.5$$

15) Expanded Griewank's plus Rosenbrock's Function

$$f_{23}(\mathbf{x}) = f_7(f_4(x_1, x_2)) + f_7(f_4(x_2, x_3)) + ... + f_7(f_4(x_{D-1}, x_D)) + f_7(f_4(x_D, x_1))$$

16) Expanded Scaffer's F6 Function

Scaffer's F6 Function: $g(x, y) = 0.5 + \dfrac{(\sin^2(\sqrt{x^2 + y^2}) - 0.5)}{(1 + 0.001(x^2 + y^2))^2}$

$$f_{24}(\mathbf{x}) = g(x_1, x_2) + g(x_2, x_3) + ... + g(x_{D-1}, x_D) + g(x_D, x_1)$$

1.4 Definitions of the CEC'15 Multi-Niche Test Suite

A. Expanded Scalable Function

1) Shifted and Rotated Expanded Two-Peak Trap

$$F_1(\mathbf{x}) = f_1(\mathbf{M}_1(\mathbf{x} - \boldsymbol{o}_1)) + F_1{}^*$$

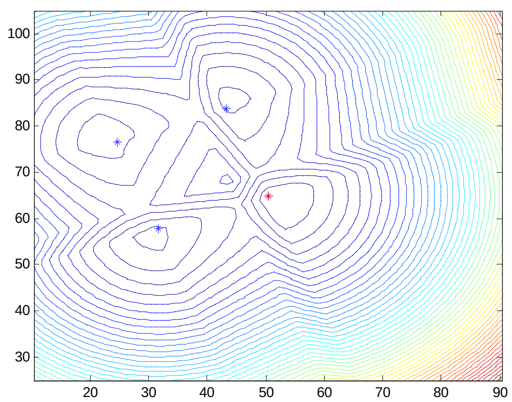

Figure 8. Contour map for 2-D function $F_1(\mathbf{x})$

Properties:

- One global optimum with 2^D-1local optima
- Non-separable

2) Shifted and Rotated Expanded Five-Uneven-Peak Trap

$$F_2(\boldsymbol{x}) = f_2(\mathbf{M}_2(\boldsymbol{x}-\boldsymbol{o}_2)) + F_2*$$

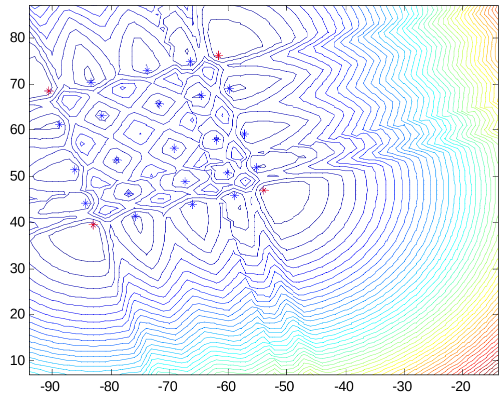

Figure 9.Contour map for 2-D function $F_2(\boldsymbol{x})$

Properties:

- 2^D global optima with 5^D-2^D local optima
- Non-separable

3) Shifted and Rotated Expanded Equal Minima

$$F_3(\boldsymbol{x}) = f_3(\mathbf{M}_3(\frac{\boldsymbol{x}-\boldsymbol{o}_3}{20})) + F_3*$$

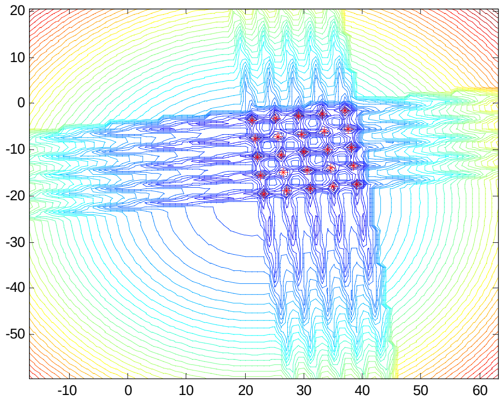

Figure 10.Contour map for 2-D function $F_3(\boldsymbol{x})$

Properties:

- 5^D global optima
- Non-separable

4) Shifted and Rotated Expanded Decreasing Minima

$$F_4(\boldsymbol{x}) = f_4(\mathbf{M}_4(\frac{\boldsymbol{x}-\boldsymbol{o}_4}{20})) + F_4*$$

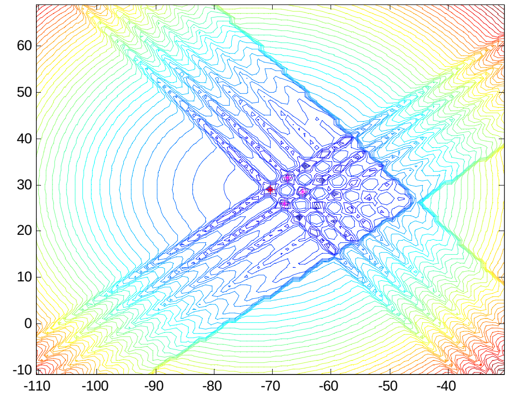

Figure 11.Contour map for 2-D function $F_4(\boldsymbol{x})$

Properties:

- One global optima with 5^D-1 local optima
- Non-separable

5) Shifted and Rotated Expanded Uneven Minima

$$F_5(\boldsymbol{x}) = f_5(\mathbf{M}_5(\frac{\boldsymbol{x}-\boldsymbol{o}_5}{20})) + F_5*$$

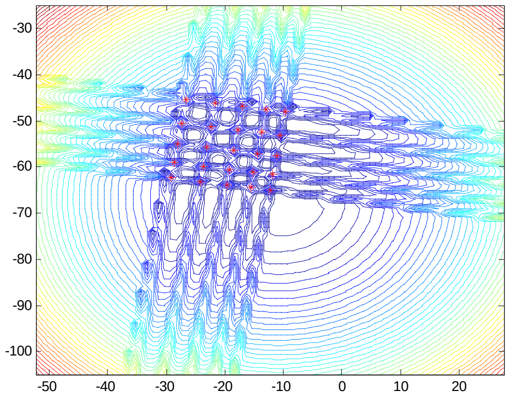

Figure 12.Contour map for 2-D function $F_5(\boldsymbol{x})$

Properties:

- 5^D global optima
- Non-separable

6) Shifted and Rotated Expanded Himmelblau's Function

$$F_6(\boldsymbol{x}) = f_6(\mathbf{M}_6(\frac{\boldsymbol{x}-\boldsymbol{o}_6}{5})) + F_6{}^*$$

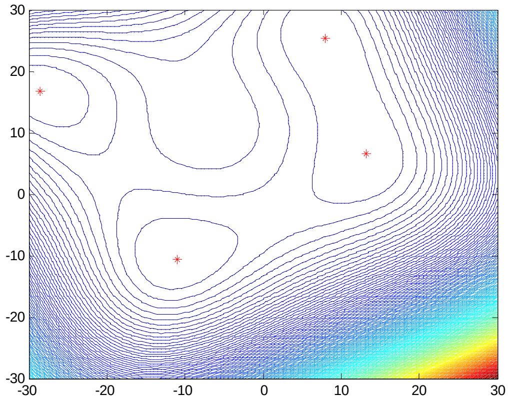

Figure 13.Contour map for 2-D function $F_6(\boldsymbol{x})$

Properties:

- $4^{D/2}$ global optima
- Non-separable

7) Shifted and Rotated Expanded Six-Hump Camel Back

$$F_7(\boldsymbol{x}) = f_7(\mathbf{M}_7(\boldsymbol{x}-\boldsymbol{o}_7)) + F_7{}^*$$

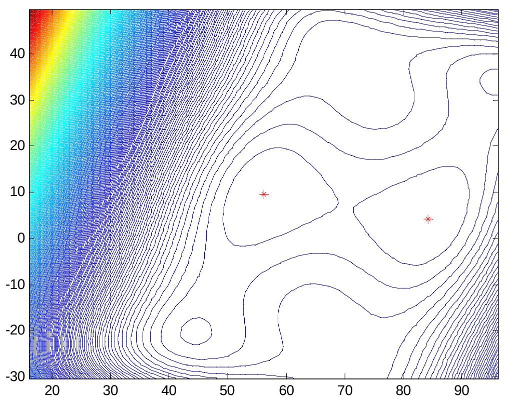

Figure 14.Contour map for 2-D function $F_7(\boldsymbol{x})$

Properties:

- $2^{D/2}$ global optima
- Non-separable

8) **Shifted and Rotated Expanded Six-Hump Camel Back**

$$F_8(\mathbf{x}) = f_8(\mathbf{M}_8(\frac{\mathbf{x}-\mathbf{o}_8}{5})) + F_8*$$

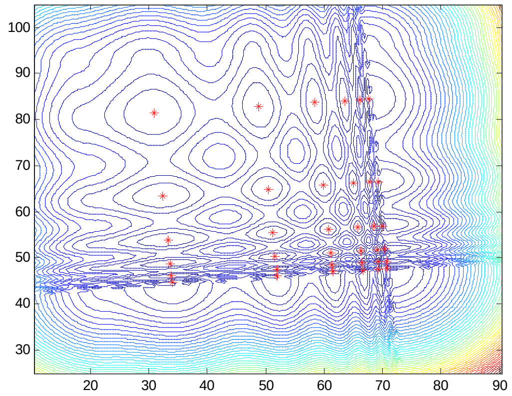

Figure 15.Contour map for 2-D function $F_8(\mathbf{x})$

Properties:

- 6^D global optima
- Non-separable

B. Composition Function

$$F(\mathbf{x}) = \sum_{i=1}^{N} \{\omega_i * [\lambda_i g_i(\mathbf{x}) + bias_i]\} + F*$$

$F(\mathbf{x})$: composition function

$g_i(\mathbf{x})$: i^{th} basic function used to construct the composition function

N: number of basic functions

o_i: new shifted optimum position for each $g_i(\mathbf{x})$, define the global and local optima's position

$bias_i$: defines which optimum is global optimum

σ_i : used to control each $g_i(\mathbf{x})$'s coverage range, a small σ_i give a narrow range for that $g_i(x)$

λ_i : used to control each $g_i(\mathbf{x})$'s height

w_i : weight value for each $g_i(\mathbf{x})$, calculated as below:

$$w_i = \frac{1}{\sqrt{\sum_{j=1}^{D}(x_j - o_{ij})^2}} \exp\left(-\frac{\sum_{j=1}^{D}(x_j - o_{ij})^2}{2D\sigma_i^2}\right)$$

Then normalize the weight $\omega_i = w_i / \sum_{i=1}^{n} w_i$

So when $\boldsymbol{x} = \boldsymbol{o}_i$, $\omega_j = \begin{cases} 1 & j = i \\ 0 & j \neq i \end{cases}$ for $j = 1, 2, ..., N$, $f(x) = bias_i + f^*$

The local optimum which has the smallest bias value is the global optimum. The composition function merges the properties of the sub-functions better and maintains continuity around the global/local optima.

Functions $Fi' = Fi - F_i^*$ are used as g_i. In this way, the function values of global optima of g_i are equal to 0 for all composition functions in this report.

Fot some composition functions, the hybrid functions are also used as the basic functions. With hybrid functions as the basic functions, the composition function can have different properties for different variables subcomponents.

9) Composition Function 1

$N = 10$

σ = [10, 20, 10, 20, 10, 20, 10, 20, 10, 20]

λ = [1, 1, 1e-6, 1e-6, 1e-6, 1e-6, 1e-4, 1e-4, 1e-5, 1e-5]

$bias$ =[0, 0, 0, 0, 0, 0, 0, 0, 0, 0] +F_8*

g_{1-2}: Rotated Sphere Function

$$g_i(\boldsymbol{x}) = f_9(\mathbf{M}_i(\boldsymbol{x} - \boldsymbol{o}_i)) + bias_i, \quad i=1, 2$$

g_{3-4}: Rotated High Conditioned Elliptic Function

$$g_i(\boldsymbol{x}) = f_{10}(\mathbf{M}_i(\boldsymbol{x} - \boldsymbol{o}_i)) + bias_i, \quad i=3, 4$$

g_{5-6}: Rotated Bent Cigar Function

$$g_i(\boldsymbol{x}) = f_{11}(\mathbf{M}_i(\boldsymbol{x} - \boldsymbol{o}_i)) + bias_i, \quad i=5, 6$$

g_{7-8}: Rotated Discus Function

$$g_i(\boldsymbol{x}) = f_{12}(\mathbf{M}_i(\boldsymbol{x}-\boldsymbol{o}_i)) + bias_i, \quad i=7,\ 8$$

$g_{9\text{-}10}$: Rotated Different Powers Function

$$g_i(\boldsymbol{x}) = f_{13}(\mathbf{M}_i(\boldsymbol{x}-\boldsymbol{o}_i)) + bias_i, \quad i=9,\ 10$$

Properties:

- Multi-modal
- Non-separable
- All sub-functions are uni-modal functions
- Ten global optima
- Different properties around different local optima

10) Composition Function 2

N = 10

σ = [10, 20, 30, 40, 50, 60, 70, 80, 90, 100]

λ = [1e-5, 1e-5, 1e-6, 1e-6, 1e-6, 1e-6, 1e-4, 1e-4, 1, 1]

$bias$ = [0, 10, 20, 30, 40, 50, 60, 70, 80, 90] +F_{10}*

$g_{1\text{-}2}$: Rotated High Conditioned Elliptic Function

$$g_i(\boldsymbol{x}) = f_{10}(\mathbf{M}_i(\boldsymbol{x}-\boldsymbol{o}_i)) + bias_i, \quad i=1,\ 2$$

$g_{3\text{-}4}$: Rotated Different Powers Function

$$g_i(\boldsymbol{x}) = f_{13}(\mathbf{M}_i(\boldsymbol{x}-\boldsymbol{o}_i)) + bias_i, \quad i=3,\ 4$$

$g_{5\text{-}6}$: Rotated Bent Cigar Function

$$g_i(\boldsymbol{x}) = f_{14}(\mathbf{M}_i(\boldsymbol{x}-\boldsymbol{o}_i)) + bias_i, \quad i=5,\ 6$$

$g_{7\text{-}8}$: Rotated Discus Function

$$g_i(\boldsymbol{x}) = f_{12}(\mathbf{M}_i(\boldsymbol{x}-\boldsymbol{o}_i)) + bias_i, \quad i=7,\ 8$$

$g_{9\text{-}10}$: Rotated Sphere Function

$$g_i(\boldsymbol{x}) = f_{9}(\mathbf{M}_i(\boldsymbol{x}-\boldsymbol{o}_i)) + bias_i, \quad i=9,\ 10$$

Properties:

- Multi-modal

- Non-separable
- All sub-functions are unimodal functions
- 1 global optimum and nine local optima
- The better optimum has a narrower region
- Different properties around different local optima

11) Composition Function 3

$N = 10$

σ = [10, 10, 10, 10, 10, 10, 10, 10, 10, 10]

λ = [0.1, 0.1, 10, 10, 10, 10, 100, 100, 1, 1]

$bias$ = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0] +F_{11}*

$g_{1\text{-}2}$: Rotated Rosenbrock's Function

$$g_i(\boldsymbol{x}) = f_{14}(\mathbf{M}_i \frac{2.048(\boldsymbol{x}-\boldsymbol{o}_i)}{100}+1)+bias_i, \quad i=1, 2$$

$g_{3\text{-}4}$: Rotated Rastrigin's Function

$$g_i(\boldsymbol{x}) = f_{18}(\mathbf{M}_i \frac{5.12(\boldsymbol{x}-\boldsymbol{o}_i)}{100})+bias_i, \quad i=3, 4$$

$g_{5\text{-}6}$: Rotated HappyCat Function

$$g_i(\boldsymbol{x}) = f_{21}(\mathbf{M}_i \frac{5(\boldsymbol{x}-\boldsymbol{o}_i)}{100})+bias_i, \quad i=5, 6$$

$g_{7\text{-}8}$: Rotated Scaffer's F6 Function

$$g_i(\boldsymbol{x}) = f_{24}(\mathbf{M}_i(\boldsymbol{x}-\boldsymbol{o}_i))+bias_i, \quad i=7, 8$$

$g_{9\text{-}10}$: Rotated Expanded Modified Schwefel's Function

$$g_i(\boldsymbol{x}) = f_{19}(\mathbf{M}_i \frac{1000(\boldsymbol{x}-\boldsymbol{o}_i)}{100})+bias_i, \quad i=9, 10$$

Properties:

- Multi-modal
- Non-separable
- All sub-functions are multimodal functions
- Ten global optima and many local optima

12) Composition Function 4

$N = 10$

σ = [10, 10, 20, 20, 30, 30, 40, 40, 50, 50]

λ = [0.1, 0.1, 10, 10, 10, 10, 100, 100, 1, 1]

$bias$ = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0] +F_{12}*

$g_{1\text{-}2}$: Rotated Rosenbrock's Function

$$g_i(\boldsymbol{x}) = f_{14}(\mathbf{M}_i \frac{2.048(\boldsymbol{x}-\boldsymbol{o}_i)}{100}+1)+bias_i, \quad i=1, 2$$

$g_{3\text{-}4}$: Rotated Rastrigin's Function

$$g_i(\boldsymbol{x}) = f_{15}(\mathbf{M}_i \frac{5.12(\boldsymbol{x}-\boldsymbol{o}_i)}{100})+bias_i, \quad i=3, 4$$

$g_{5\text{-}6}$: Rotated HappyCat Function

$$g_i(\boldsymbol{x}) = f_{21}(\mathbf{M}_i \frac{5(\boldsymbol{x}-\boldsymbol{o}_i)}{100})+bias_i, \quad i=5, 6$$

$g_{7\text{-}8}$: Rotated Scaffer's F6 Function

$$g_i(\boldsymbol{x}) = f_{24}(\mathbf{M}_i(\boldsymbol{x}-\boldsymbol{o}_i))+bias_i, \quad i=7, 8$$

$g_{9\text{-}10}$: Rotated Expanded Modified Schwefel's Function

$$g_i(\boldsymbol{x}) = f_{19}(\mathbf{M}_i \frac{1000(\boldsymbol{x}-\boldsymbol{o}_i)}{100})+bias_i, \quad i=9, 10$$

Properties:

- Multi-modal
- Non-separable
- All sub-functions are multimodal functions
- Ten global optima and many local optima
- The better optimum has a narrower region

13) Composition Function 5

$N = 10$

σ = [10, 20, 30, 40, 50, 60, 70, 80, 90, 100]

λ = [0.1, 10, 10, 0.1, 2.5, 1e-3, 100, 2.5, 10, 1]

$bias$ = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0] +F_{13}*

g_1: Rotated Rosenbrock's Function

$$g_1(\boldsymbol{x}) = f_{14}(\mathbf{M}_1 \frac{2.048(\boldsymbol{x}-\boldsymbol{o}_1)}{100}+1)+bias_1$$

g_2: Rotated HGBat Function

$$g_2(\boldsymbol{x}) = f_{22}(\mathbf{M}_2 \frac{5(\boldsymbol{x}-\boldsymbol{o}_2)}{100})+bias_2$$

g_3: Rotated Rastrigin's Function

$$g_3(\boldsymbol{x}) = f_{18}(\mathbf{M}_3 \frac{5.12(\boldsymbol{x}-\boldsymbol{o}_3)}{100})+bias_3$$

g_4: Rotated Ackley's Function

$$g_4(\boldsymbol{x}) = f_{15}(\mathbf{M}_4(\boldsymbol{x}-\boldsymbol{o}_4))+bias_4$$

g_5: Rotated Weierstrass Function

$$g_5(\boldsymbol{x}) = f_{16}(\mathbf{M}_5 \frac{0.5(\boldsymbol{x}-\boldsymbol{o}_5)}{100})+bias_5$$

g_6: Rotated Katsuura Function

$$g_6(\boldsymbol{x}) = f_{20}(\mathbf{M}_6 \frac{5(\boldsymbol{x}-\boldsymbol{o}_6)}{100})+bias_6$$

g_7: Rotated Scaffer's F6 Function

$$g_7(\boldsymbol{x}) = f_{24}(\mathbf{M}_7(\boldsymbol{x}-\boldsymbol{o}_7))+bias_7$$

g_8: Rotated Expanded Griewank's plus Rosenbrock's Function

$$g_8(\boldsymbol{x}) = f_{23}(\mathbf{M}_8 \frac{5(\boldsymbol{x}-\boldsymbol{o}_8)}{100})+bias_8$$

g_9: Rotated HappyCat Function

$$g_9(\boldsymbol{x}) = f_{21}(\mathbf{M}_9 \frac{5(\boldsymbol{x}-\boldsymbol{o}_9)}{100})+bias_9$$

g_{10}: Rotated Expanded Modified Schwefel's Function

$$g_{10}(\boldsymbol{x}) = f_{19}(\mathbf{M}_{10} \frac{1000(\boldsymbol{x}-\boldsymbol{o}_{10})}{100})+bias_{10}$$

Properties:

- Multi-modal
- Non-separable
- All sub-functions are multimodal functions
- Ten global optima and many local optima
- The better optimum has a narrower region

C. Niching Optimization based on Distance among Optima

In this part, the required number of goal optima is provided while the exact positions of these optima are not provided. The participants are required to search for the optima based on the distance among optima. The Euclidean distance among the final obtained optima should not be smaller than the predefined value. The average quality of the obtained solutions is used to rank the algorithms.

14) Composition Function6

$N = 10$

$\sigma = [10, 10, 20, 20, 30, 30, 40, 40, 50, 50]$

$\lambda = [10, 1, 10, 1, 10, 1, 10, 1, 10, 1]$

$bias = [0, 20, 40, 60, 80, 100, 120, 140, 160, 180] + F_{14}*$

$g_{1,3,5,7,9}$: Rotated Rastrigin's Function

$$g_i(\boldsymbol{x}) = f_{18}(\mathbf{M}_i \frac{5.12(\boldsymbol{x}-\boldsymbol{o}_i)}{100}) + bias_i, \quad i = 1, 3, 5, 7, 9$$

$g_{2,4,6,8,10}$: Rotated Expanded Modified Schwefel's Function

$$g_i(\boldsymbol{x}) = f_{19}(\mathbf{M}_i \frac{1000(\boldsymbol{x}-\boldsymbol{o}_i)}{100}) + bias_i, \quad i = 2, 4, 6, 8, 10$$

Properties:

- Multi-modal
- Non-separable
- All sub-functions are multimodal functions
- One global optimum and many local optima
- The better optimum has a narrower region
- Predefined optima distance: Dis_{10D}=113, Dis_{20D} =183, Dis_{30D} =285

15) Composition Function 7

$N = 10$

$\sigma = [10, 20, 30, 40, 50, 60, 70, 80, 90, 100]$

$\lambda = [0.1, 10, 10, 0.1, 2.5, 1e\text{-}3, 100, 2.5, 10, 1]$

$bias = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0] + F_{15}*$

g_1: Rotated Rosenbrock's Function

$$g_1(\boldsymbol{x}) = f_{14}(\mathbf{M}_1 \frac{2.048(\boldsymbol{x}-\boldsymbol{o}_1)}{100}+1)+bias_1$$

g_2: Rotated HGBat Function

$$g_2(\boldsymbol{x}) = f_{22}(\mathbf{M}_2 \frac{5(\boldsymbol{x}-\boldsymbol{o}_2)}{100})+bias_2$$

g_3: Rotated Rastrigin's Function

$$g_3(\boldsymbol{x}) = f_{18}(\mathbf{M}_3 \frac{5.12(\boldsymbol{x}-\boldsymbol{o}_3)}{100})+bias_3$$

g_4: Rotated Ackley's Function

$$g_4(\boldsymbol{x}) = f_{15}(\mathbf{M}_4(\boldsymbol{x}-\boldsymbol{o}_4))+bias_4$$

g_5: Rotated Weierstrass Function

$$g_5(\boldsymbol{x}) = f_{16}(\mathbf{M}_5 \frac{0.5(\boldsymbol{x}-\boldsymbol{o}_5)}{100})+bias_5$$

g_6: Rotated Katsuura Function

$$g_6(\boldsymbol{x}) = f_{20}(\mathbf{M}_6 \frac{5(\boldsymbol{x}-\boldsymbol{o}_6)}{100})+bias_6$$

g_7: Rotated Scaffer's F6 Function

$$g_7(\boldsymbol{x}) = f_{24}(\mathbf{M}_7(\boldsymbol{x}-\boldsymbol{o}_7))+bias_7$$

g_8: Rotated Expanded Griewank's plus Rosenbrock's Function

$$g_8(\boldsymbol{x}) = f_{23}(\mathbf{M}_8 \frac{5(\boldsymbol{x}-\boldsymbol{o}_8)}{100})+bias_8$$

g_9: Rotated HappyCat Function

$$g_9(\boldsymbol{x}) = f_{21}(\mathbf{M}_9 \frac{5(\boldsymbol{x}-\boldsymbol{o}_9)}{100})+bias_9$$

g_{10}: Rotated Expanded Modified Schwefel's Function

$$g_{10}(\boldsymbol{x}) = f_{19}(\mathbf{M}_{10} \frac{1000(\boldsymbol{x}-\boldsymbol{o}_{10})}{100})+bias_{10}$$

Properties:

- Multi-modal
- Non-separable
- All sub-functions are multimodal functions
- One global optimum and many local optima
- The better optimum has a narrower region

- Predefined optima distance: Dis_{10D}=139, Dis_{20D} =191, Dis_{30D} =301

2. Evaluation Criteria

2.1 Experimental Setting

Problems: 15 minimization problems

Dimensions: Refer to Table I

Runs / problem: 51 (**Do not run many 51 runs to pick the best run**)

MaxFES: $2000 * D * \sqrt{q}$. Here q is the goal optima number. (For example, for 5D function 1, q=4, MaxFES=2000*5*2=20000).

Search Range: $[-100,100]^D$

Initialization: Uniform random initialization within the search space. Random seed is based on time, Matlab users can use rand('state', sum(100*clock)).

Global Optima: All problems have the required number of optima within the given bounds and it is NOT allowed to perform search outside of the given bounds for these problems, as solutions outside of the bounds are regarded as infeasible.

$F_i(\boldsymbol{x}^*) = F_i(\boldsymbol{o}_i) = F_i^*$

Termination: Terminate when reaching MaxFES or the error value is smaller than 10^{-8}.

2.2 Performance Metric

1) Success Rates (SR)

Success rate is the percentage of runs in which all the desired peaks are successfully located. The level of accuracy is set to 0.1 in this competition. This parameter is used to measure how close the obtained solutions are to the known global/local peaks. If a solution is obtained which is within a distance to the actual solution which is lower than a tolerance value (level of accuracy), the optimal solution is considered to have been found. If all the desired peaks are found in one single run, this run is considered to be successful.

2) Average Number of Optima Found (ANOF)

This criterion is used to compare the average number of peaks found over 51 runs using the given level of accuracy.

3) Success Performance (SP)

The success performance is calculated using the following equation:

$$Success\ performance = \frac{Average\ number\ of\ \ function\ evaluations}{success\ rate}$$

Note that the success performance can be obtained only when the success rate is not zero. An algorithm consuming less function evaluations and yielding higher success rate is considered better. Hence, smaller values of success performance are desirable.

4) Maximum Peak Ratio Statistic (MPR)

To test the quality of optima without considering the distribution of the population, the performance metric called the maximum peak ratio statistic (MPR) is adopted. The maximum peak ratio is defined as follows (assuming a minimization problem):

$$MPR = \frac{\sum_{i=1}^{q}(F_i - F^* + 1)}{\sum_{i=1}^{q}(f_i - F^* + 1)}$$

where q is the number of optima, $\{f_i\}_{i=1}^{q}$ are the fitness values of the optima in the final population, $\{F_i\}_{i=1}^{q}$ are the values of real optima of the objective function while F^* is the function value of the global optimum. All the values are assumed to be positive. Note that a larger MPR value indicates a better performance of the algorithm.

The performance of each algorithm depends on the specified level of accuracy. Note that for more challenging functions with a tight level of accuracy, if no run is able to find all peaks, the success rate will be zero.

2.3 Results Record

1) For functions 1-13, calculate **Success Rates, Average Number of Optima Found, Success Performance and Maximum Peak Ratio Statistic** according to section 2.2 and present the **best, worst, mean, median** and **standard variance** values of these four performance metrics for the 51 runs.

2) For functions 14-15, since the participants are required to search for the optima based on the distance among optima and the exact positions of these goal optima are not provided, the performance metrics cannot be calculated. The **average values** of the error values of achieved **best, worst, median** optima and the corresponding **standard deviation** values of these for the 51 runs are require to recorded.

3) Algorithm Complexity

a) Run the test program below:

for i=1:1000000

x= 0.55 + (double) i;

$x=x + x$; $x=x/2$; $x=x*x$; $x=sqrt(x)$; $x=log(x)$; $x=exp(x)$; $x=x/(x+2)$;

end

Computing time for the above=*T0*;

b) Evaluate the computing time just for **Function 13**. For 200000 evaluations of a certain dimension *D*, it gives *T1*;

c) The complete computing time for the algorithm with 200000 evaluations of the same *D* dimensional **Function 13** is *T2*.

d) Execute step c **five** times and get **five** *T2* values. $\widehat{T}2$ =Mean(*T2*)

The complexity of the algorithm is reflected by: $\widehat{T}2$, *T1*, *T0*, and ($\widehat{T}2$ -*T1*)/*T0*

The algorithm complexities are calculated on 10, 20, 30 dimensions, to show the algorithm complexity's relationship with dimension. Also provide sufficient details on the computing system and the programming language used. In step c, we execute the complete algorithm **five** times to accommodate variations in execution time due adaptive nature of some algorithms.

Please Note: Similar programming styles should be used for all *T0*, *T1* and *T2*.

(For example, if *m* individuals are evaluated at the same time in the algorithm, the same style should be employed for calculating *T1*; if parallel calculation is employed for calculating *T2*, the same way should be used for calculating *T0* and *T1*. In other word, the complexity calculation should be fair.)

4) Parameters

Participants must not search for a distinct set of parameters for each problem/dimension/etc.

Please provide details on the following whenever applicable:

a) All parameters to be adjusted

b) Corresponding dynamic ranges

c) Guidelines on how to adjust the parameters

d) Estimated cost of parameter tuning in terms of number of FEs

e) Actual parameter values used.

5) Encoding

If the algorithm requires encoding, then the encoding scheme should be independent of the specific problems and governed by generic factors such as the search ranges.

5) Results Format

The participants are required to send the final results as the required format to the organizers and the organizers will present an overall analysis and comparison based on these results.

a) Record the obtained q solutions and the corresponding function error value ($F_i(\boldsymbol{x})$-$F_i(\boldsymbol{x}^*)$) at MaxFES for each run.

Create one txt document with the name "AlgorithmName_FunctionNo._D_RunsNo..txt" for each run.

For example, PSO results for test function 5 and D=30, the file name for the first run should be "PSO_5_30_1.txt".

Then save the results matrix (***the gray shadowing part, a*** **(*D*+*1*)*****q matrix***) as Table II in the file. Thus there should be **51 txt files** for each function of a certain dimension

Table IX. Results Matrix for D Dimensional Function X of $^{i\text{th}}$ run

$f(\boldsymbol{x}_1)$	x_{11}	x_{12}	...	x_{1D}
$f(\boldsymbol{x}_1)$	x_{21}	x_{22}	...	x_{2D}
...	...	...	...	...
$f(\boldsymbol{x}_q)$	x_{q1}	x_{q2}	...	x_{qD}

Please Note:

1. Error value smaller than 10^{-8} will be taken as zero. Predefined level of accuracy = 0.1.

2. Please check the solutions to make sure they are in the predefined search range. Final solutions out of the bound are not acceptable.

b) FES used for finding each optimum (satisfying the predefined level of accuracy).

In this case, *q* FES values are recorded for each function for each run. If the optimum is not found in the end of the run, record FES=Inf in the results.

Create one txt document with the name "AlgorithmName_FunctionNo._D_FES..txt" for each run.

For example, PSO results for test function 5 and *D*=30, the file name for the first run should be "PSO_5_30_FES.txt".

Then save the results matrix (***the gray shadowing part, a* 51*q *matrix***) as Table II in the file. Thus there should be **one txt file** for each function of a certain dimension. With the 51 error matrix mentioned in (a), ***52 txt files* are required to submitted to the organizer for a function of a certain dimension in total.**

Table X. FES matrix for *D* Dimensional Function X

***FES.txt	1	2	…	*q*
Run 1				
Run 2				
...				
Run 51				

Notice: All participants are allowed to improve their algorithms further after submitting the initial version of their papers to CEC2015. And they are required to submit their results in the introduced format to the organizers after submitting the **final** version of paper as soon as possible.

2.3 Results Temple

Language: Matlab 2008a

Algorithm: Particle Swarm Optimizer (PSO)

<u>Results</u>

Table XI. Success Rates (Functions 1-13)

Func.	Dimension	Best	Worst	Median	Mean	Std
	5					
1	10					
	20					

2	2					
	5					
	8					
3	2					
	3					
	4					
4	5					
	10					
	20					
5	2					
	3					
	4					
6	4					
	6					
	8					
7	6					
	10					
	16					
8	10					
	20					
	30					
9	10					
	20					
	30					
10	10					
	20					
	30					
11	10					
	20					
	30					
12	10					
	20					
	30					
13	10					
	20					
	30					

Table XII. Average Number of Optima Found (Functions 1-13)

Func.	**Dimension**	**Best**	**Worst**	**Median**	**Mean**	**Std**
1	5					
	10					
	20					
	2					

2	5					
	8					
3	2					
	3					
	4					
4	5					
	10					
	20					
5	2					
	3					
	4					
6	4					
	6					
	8					
7	6					
	10					
	16					
8	10					
	20					
	30					
9	10					
	20					
	30					
10	10					
	20					
	30					
11	10					
	20					
	30					
12	10					
	20					
	30					
13	10					
	20					
	30					

Table XIII. Success Performance (Functions 1-13)

Func.	Dimension	Best	Worst	Median	Mean	Std
1	5					
	10					
	20					
	2					

2	5
	8
3	2
	3
	4
4	5
	10
	20
5	2
	3
	4
6	4
	6
	8
7	6
	10
	16
8	10
	20
	30
9	10
	20
	30
10	10
	20
	30
11	10
	20
	30
12	10
	20
	30
13	10
	20
	30

Table XIV. Maximum Peak Ratio Statistic (Functions 1-13)

Func.	**Dimension**	**Best**	**Worst**	**Median**	**Mean**	**Std**
1	5					
	10					
	20					
	2					

2	5
	8
3	2
	3
	4
4	5
	10
	20
5	2
	3
	4
6	4
	6
	8
7	6
	10
	16
8	10
	20
	30
9	10
	20
	30
10	10
	20
	30
11	10
	20
	30
12	10
	20
	30
13	10
	20
	30

Table XV. Error Values for Functions 14-15

Func.	**Dimension**		**Best**	**Worst**	**Median**	**Mean**	**Std**
14	10	Mean					
		Std.					
	20	Mean					
		Std.					

	30	Mean	
		Std.	
15	10	Mean	
		Std.	
	20	Mean	
		Std.	
	30	Mean	
		Std.	

Algorithm Complexity

Table XVI. Computational Complexity

	T0	*T1*	$\hat{T}2$	($\hat{T}2$ -*T1*)/*T0*
D=10				
D=20				
D=30				

Parameters

a) All parameters to be adjusted

b) Corresponding dynamic ranges

c) Guidelines on how to adjust the parameters

d) Estimated cost of parameter tuning in terms of number of FES

e) Actual parameter values used.

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